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THE UNIVERSITY OF ALBERTA

EFFECT OF CONFINEMENT PRESSURE ON VISCO-INERTIAL FLOW PARAMETERS IN POROUS MEDIA

by

Yasin Senturk (C)

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled "Effect of Confinement Pressure on Visco-Inertial Flow Parameters in Porous Media" submitted by Yasin Senturk in partial fulfilment of the requirements for the degree of Master of Science in Petroleum Engineering.

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Date September 29, 1975.

ABSTRACT

An existing definition of rock confinement pressure has been modified to account for the design features of the flow cell. Use of the modified definition (to experimental work) reduces the magnitude of the stress created on the rock sample.

Using the modified rock confinement pressure, flow tests under different net confinement pressures were conducted on three Indiana limestone and two Blairmore sandstone samples in order to determine the effect of confinement pressure on the visco-inertial flow parameters K, b, and F_B. The observed flow data were used to estimate these parameters both graphically and numerically. The non-linear least squares parameter estimation technique, employing the Gauss-Newton algorithm, was found superior to that of the graphical method which requires data splitting. The effect of confinement pressure on the parameters, as determined in this study, compares favorably with that reported in the literature for linear systems.

The sensitivity of these parameters to the number of data points, i.e. points in the viscous and/or inertial regions, was also studied.

Recent studies imply that the inertial resistance coefficient, F_B, may be a property of the fluid as well as the rock. Using nitrogen, argon and methane, flow tests were conducted on two rock samples, the results of which indicate a weak dependence of F_B upon gas properties.

Some anomalous behavior of the Klinkenberg plots was observed in this study. It is believed that the peculiarities related to the runs employing the flow test method Case I, were caused by inadequate confinement of the rock sample which in turn caused gas to flow in the interfaces as well as through the rock. However, it is felt that the available information is insufficient at this time to explain the behavior of the Klinkenberg plots for those runs employing the test method Case II, and that further detailed research is required.

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TABLE OF CONTENTS

CHAPTER		PAGE
I	INTRODUCTION	1
II	LITERATURE SURVEY	3
	A. Viscous Fluid Flow Through Porous Media	3
	B. Visco-Inertial Fluid Flow Through Porous Media	5
	C. Correlations for b and F_B	12
III	THEORY	14
	A. Visco-Inertial Flow Theory	14
	1. Differential Model	16
	2. Integrated Model	16
	B. Turbulent Flow Theory	17
	C. Concept of Rock Confinement Pressure	18
	1. Net Confinement Pressure	18
	2. Proposed Net Confinement Pressure	19
IV	PARAMETER ESTIMATION TECHNIQUES	23
	A. Numerical Parameter Estimation Technique: The Gauss-Newton Method	23
	B. Graphical Parameter Estimation Techniques	24
	1. A Graphical Estimation Technique	25
	(a) The Klinkenberg Plot	25
	(b) Modified Visco-Inertial Plot	26

CHAPTER PAGE

2. A Proposed Semi-Graphical Parameter Estimation Technique 26

V EXPERIMENTATION 29

A. Experimental Apparatus 29

B. Experimental Procedures 29

1. Preparation of Rock Samples 29

2. Operational Procedure for Radial Gas Flow Equipment 33

3. Flow Test Methods 34

VI TREATMENT OF DATA 36

A. Evaluation of Fluid Properties 36

1. Gas Compressibility Factor 36

2. Gas Viscosity 37

B. Evaluation of Results 39

VII RESULTS AND DISCUSSION 40

A. Net Confinement Pressure 40

B. Parameter Estimation 45

1. Choice of Parameter Estimation Technique 45

2. Sensitivity Analysis 49

C. Inertial Coefficient as a Property of the Rock and the Fluid 51

D. Effect of Confinement Pressure on Parameters . . 57

1. Problems Encountered 57

(a) Runs with Flow Test Method Case I 57

(b) Runs with Flow Test Method Case II 71

CHAPTER PAGE

2. Confinement Pressure Effect 74

VIII CONCLUSIONS AND RECOMMENDATIONS 81

A. Conclusions 81

B. Recommendations 82

NOMENCLATURE 84

BIBLIOGRAPHY 89

APPENDICES 96

APPENDIX A: Development of the Uberoi Flow Equation . . . 97

APPENDIX B: Application of the Numerical and the Semi-Graphical Parameter Estimation Techniques to the Integrated Model of Visco-Inertial Flow Theory 102

APPENDIX C: Tabulated Raw Experimental Data 131

APPENDIX D: Tabulated Processed Data and Results of Parameter Estimation Techniques 160

LIST OF TABLES

TABLE		PAGE
1	Summary of Results for Runs 1 and 2 on Sample LS-2	44
2	Physical Properties of the Rock Samples	47
3	Results of Data Discrimination Methods	48
4	Estimated Values of K, b and F_B for Sample SS-2-A Run 2	50
5	Sensitivity of K, b and F_B to the Number of Data Points	51
6	Summary of Results for Runs with Different Gases	54
7	Fluid Properties and Estimated Values of b and F_B	56
8	Errors in Flow Rate Measurements	73
9	Values of K, b and F_B for the Rock Samples	75

LIST OF FIGURES

FIGURE		PAGE
1	Radial Flow Cell and Free Body Diagram of Forces on the Piston	20
2	A Flow System Layout for the Porosimeter	30
3	A Complete Layout of the Radial Gas Flow Apparatus	31
4	Radial Flow Cell	32
5	P_e Versus P_{hc} Type Curves for Flow Test Method Case I	41
6	P_w Versus P_{hc} Type Curves for Flow Test Method Case II	43
7	Klinkenberg Plots for Sample LS-2 Runs 1 and 2	46
8	Klinkenberg Plots for Sample LS-1 Runs 1A, 1B and 1C	52
9	Klinkenberg Plots for Sample SS-2-A Runs 1A, 1B and 1C	53
10	F_B as a Function of μ	58
11	F_B as a Function of G	59
12	Klinkenberg Plot for Sample LS-1 Run 2	60
13	Klinkenberg Plot for Sample LS-1 Run 3	61
14	Klinkenberg Plot for Sample LS-1 Run 4	62
15	Klinkenberg Plots for Sample LS-3-A Runs 1, 2, 3 and 4	63
16	Klinkenberg Plots for Sample SS-2-A Runs 1A, 2, 3 and 4	64
17	Klinkenberg Plots for Sample LS-2 Runs 2, 3, 4 and 5	65

FIGURE		PAGE
18	Klinkenberg Plots for Sample SS-1-A Runs 1, 2, and 3	66
19	Klinkenberg Plots for Sample LS-1 Runs 5 and 6 . . .	67
20	Confining Pressure Effect on K	76
21	Confining Pressure Effect on b	77
22	Confining Pressure Effect on F_B	79
23	Confining Pressure Effect on F_B	80

LIST OF PLATES

PLATE		PAGE
1	Photographs of Rubber Gaskets used for Rock Sample Confinement	69

CHAPTER I

INTRODUCTION

Considerable theoretical and experimental work has been undertaken on the subject of fluid flow through porous media. It is common in the literature to describe fluid flow behavior in the viscous and visco-inertial flow regions by means of Darcy's Law and the Forchheimer equation respectively (1,2).

In his work, Piplapure (3) studied steady radial gas flow through porous media, both theoretically and experimentally. He proposed an equation to describe steady radial gas flow through porous media characterized by the viscous coefficient F_A, the inertial coefficient F_B, and the slippage coefficient b. He observed instances of peculiar behavior of the Klinkenberg plots for certain runs (4). It may be that the concept of net rock confinement pressure used in his work was the cause for this anomalous behavior.

Crafton (5) proposed radial flow equations similar to the Forchheimer quadratic flow equation by assuming the existence of steady or pseudo-steady and isotropic-turbulent flow in porous media. By including the fluid viscosity in the definition of inertial coefficient, which he designates by B, Crafton (6) assumed that F_B is a property of the fluid as well as the rock. This dependence of F_B on fluid properties was also suspected by Carman (7) when he stated that "unlike F_A, the factor F_B is not a true specific constant, but is generally a function of the flow rate q."

With these fundamental ideas in mind, it was decided to undertake an experimental study of steady radial gas flow through consolidated rock samples in order to achieve the following objectives:

1. To reexamine the concept of net rock confinement pressure.
2. To examine the possibility that the parameters K, b, and F_B are dependent upon the magnitude of the net rock confinement pressure.
3. To determine whether F_B is dependent upon fluid properties as well as rock properties.

To accomplish the first two objectives, it was decided to investigate a new definition of the net rock confinement pressure which includes the design features of the sample holder used by Piplapure. Secondly, it was decided to conduct various flow tests both on the limestone samples similar to the ones used by Piplapure and on new more competent sandstone samples, i.e. with high compressive strength and/or modulus of elasticity, under different net confinement pressures.

To accomplish the third objective, it was decided to conduct flow tests with different gases.

CHAPTER II

LITERATURE SURVEY

Since there is an abundance of available literature on the subject of fluid flow through porous media, the present section is subclassified in order to cope with various aspects of the topic. Hence, the survey covers the work done in the areas of viscous and visco-inertial fluid flow together with their experimental verifications.

A. Viscous Fluid Flow Through Porous Media

In 1856 Henry Darcy proposed an empirical equation for flow of water through a permeable medium (8). The so called Darcy's Law when modified to describe steady viscous linear horizontal incompressible fluid flow through a homogeneous porous medium can be written as

$$-\frac{dP}{dx} = \frac{\mu q}{K} \equiv F_A \mu q \qquad (1)$$

Hubbert derived Darcy's Law from the fundamental Navier-Stokes equation of motion thereby demonstrating its mechanistic basis (9).

Klinkenberg's work (10) showed that although Darcy's Law holds in the case of liquid flow, it does not hold for gas flow at low pressures. In his work, he observed that gas permeabilities are always higher than liquid permeabilities as calculated from Equation 1. He attributed this phenomenon to slippage and proposed

the idea that permeability in Equation 1 should be replaced by an "apparent permeability" defined as

$$K_a = K\left(1 + \frac{b}{\bar{P}}\right) \tag{2}$$

This shows the dependence of gas permeability upon the mean flowing pressure $\bar{P}$, where the slippage coefficient (11) may be defined as

$$b = \frac{4C\bar{\lambda}}{\bar{r}}\bar{P} \tag{3}$$

where $\bar{\lambda}$ is the mean free path of the flowing gas measured at $\bar{P}$, $\bar{r}$ is the mean pore radius and C is the proportionality factor which is approximately unity (12,13). Consequently, b is a characteristic of the porous medium and the particular flowing gas.

The work of many authors (14,15,16,17,18) verified and supported Klinkenberg's idea of slippage as applied to gas flow through porous media.

Considering isothermal and viscous flow of gas in a homogeneous porous medium, Al-Hussainy et al. (19) proposed the following equation for steady radial gas flow in the viscous region:

$$q_{sc} = \frac{\pi Kh\, T_{sc}[m(P_e) - m(P_w)]}{T\, P_{sc}\, ln(r_e/r_w)} \tag{4}$$

where

$$m(P) = 2\int_{P_m}^{P} \frac{P}{\mu(P)\, Z(P)}$$

Equation 4 does not include the slippage effect but it takes into account the pressure dependence of viscosity and compressibility.

B. Visco-Inertial Fluid Flow Through Porous Media

On the other hand, it was observed that Darcy's Law could not adequately describe fluid flow through porous media at high flow rates. Later, Forchheimer proposed the following quadratic equation to describe fluid flow through porous media (20):

$$-\frac{dP}{dx} = \alpha q + \beta q^2 \qquad (5)$$

where α and β were considered to be constants. This equation shows that as the velocity increases, inertial resistance losses, being dependent on the kinetic energy per unit volume of fluid, i.e. on ρq^2, eventually become predominant (21). It is useful to note that Equation 5 reduces to Darcy's Law when the inertial effects are neglected.

Out of further investigations (22,23,24,25,26), the following quadratic equation was developed to describe linear horizontal visco-inertial flow through porous media:

$$-\frac{dP}{dx} = F_A \mu q + F_B \rho q^2 \qquad (6)$$

where

F_A = viscous resistance coefficient

F_B = inertial resistance coefficient

Chwyl (27), applying an approach similar to that of Hubbert (28), derived the Forchheimer equation from the fundamental Navier-Stokes equation, thereby giving it a theoretical base. He assumed

that F_A and F_B are rock properties and therefore are constants for a given porous medium.

Greenberg and Weger (29) reported that both F_A and F_B remained constant for pressures up to 2000 psia under isothermal conditions, but permeability varied inversely with temperature.

Despite the observations of Al-Hussainy et al. (30), Mackett (31) concluded that the magnitude of errors due to the assumption of average fluid properties is not significant under normal laboratory conditions.

Combining Equation 6 with the modified gas law and utilizing average gas properties, Cornell and Katz (32) proposed the following integrated equation to describe steady linear horizontal gas flow through porous media:

$$\frac{M(P_1^2-P_2^2)}{2\bar{\mu}\bar{Z}\bar{T}RL\left(\frac{W}{A}\right)} = F_A + F_B\left(\frac{W}{A\bar{\mu}}\right) \qquad (7)$$

where $\frac{W}{A} = \rho q$ is the mass velocity per unit area.

Tek (33) developed a generalized Darcy equation, utilizing the friction factor - Reynolds number correlation, in the form

$$-\frac{dP}{dx} = \frac{1}{K}\mu q + \left(\ell_f \frac{\bar{D}_p}{\phi K}\right)\rho q^2 \qquad (8)$$

where ℓ_f is a "lithology factor" representing the particular porous medium, ϕ is the fractional porosity and $\bar{D}_p$ is the mean particle diameter. Hence, a comparison of Equations 6 and 8 suggests that

$$F_B = \ell_f\left(\frac{\bar{D}_p}{\phi K}\right) \qquad (9)$$

Blick (34) investigated the Forchheimer equation both theoretically and experimentally for linear flow systems. Applying the force-momentum balance together with the modified gas law, he proposed the following equation to describe linear horizontal isothermal steady gas flow through a capillary-orifice model:

$$-\frac{dP}{dx} = \left(\frac{2C_f N_{Re}}{\bar{D}^2 \phi A^*}\right) \mu q + \left(\frac{C_D}{2\bar{D} \phi A^*}\right) \rho q^2 \qquad (10)$$

where

$C_f = \frac{16}{N_{Re}}$, for viscous regime only

$\frac{1}{\sqrt{C_f}} = 4.0 \log_{10}(N_{Re} \sqrt{C_f}) - 0.4$, for turbulent regime (valid only for smooth pipe, i.e. $\varepsilon/d = 0$)

C_D = drag coefficient of orifice plate

$\bar{D}$ = mean pore diameter

N_{Re} = Reynolds number

and $A^* = 1 - \left(\frac{q}{\phi}\right)^2 \left(\frac{M}{ZRT}\right) \hat{=} 1.0$

A comparison of Equations 6 and 10 yields

$$F_A = \frac{2C_f N_{Re}}{\bar{D}^2 \phi A^*} \equiv \frac{1}{K} \qquad (11)$$

and $$F_B = \frac{C_D}{2\bar{D} \phi A^*} \qquad (12)$$

Because of the nature of A^*, Equation 12 suggests that F_B is dependent upon the type of rock and fluid and the rate of flow per unit area. Furthermore, Equation 12 is similar to Equation 11 in Blick's work but it does not contain ρ.

Stewart and Owens (35) reported that both slippage and inertial effects were present at high flow rates although slippage effects disappeared at very large Reynolds numbers.

Kolada (36) proposed a general equation, including viscous, inertial and the slippage effects simultaneously, to describe linear horizontal steady gas flow through porous media. He derived this equation by combining Equations 2 and 7 which can be written as

$$\frac{M(P_1^2-P_2^2)}{2\bar{\mu}\bar{Z}\bar{T}RL\left(\frac{W}{A}\right)} = \frac{1}{K\left(1+\frac{b}{\bar{P}}\right)} + F_B\left(\frac{W}{A\bar{\mu}}\right) \qquad (13)$$

Making use of the work done by Brownell and Katz (37), which provides friction factor - Reynolds number charts for porous media, Elenbaas and Katz (38) developed flow equations for radial flow systems which may be rewritten as

$$(P_e^2-P_w^2) = 812 \times 10^{-6} \frac{\bar{\mu}\bar{Z}\bar{T}W}{\bar{D}_p^2\,\phi^{n_1-n_2}\,hG} ln(r_e/r_w) \qquad (14)$$

for the laminar flow regime and

$$(P_e^2-P_w^2) = 812 \times 10^{-6} \frac{\bar{Z}\bar{T}C_fW^2}{\bar{D}_p\,\phi^{n_1}h^2G}\left(\frac{1}{r_w}-\frac{1}{r_e}\right) \qquad (15)$$

for the turbulent flow regime, where

W = mass flow rate, lbs/sec.

$\bar{D}_p$ = mean particle diameter, ft.

ϕ = porosity of bed, fraction

h = bed thickness, ft.

G = gas gravity, dimensionless

C_f = Fanning friction factor, dimensionless

and n_1 and n_2 are exponents denoted as n and m respectively in their paper.

Subsequently, others (39,40,41,42) attempted to apply the linear form of the Forchheimer equation to radial flow systems. Combining the modified gas law with this radial equivalent of the Forchheimer equation, Tek et al. (43) proposed the following integrated equation to describe steady plane radial gas flow through porous media, with the customary field units:

$$(P_e^2-P_w^2) = \frac{1424\bar{\mu}\bar{Z}\bar{T}Q_o}{hK} \ln\left(\frac{r_e}{r_w}\right) + \frac{3.1602 \times 10^{-12} F_B G \bar{Z}\bar{T} Q_o^2}{h^2}\left(\frac{1}{r_w} - \frac{1}{r_e}\right) \quad (16)$$

Piplapure (44) further generalized this equation by including the slippage effects which yielded the non-linear ordinary differential equation

$$-\frac{dP}{dr} = \frac{C_1}{r(P+b)} + \frac{C_2 F_B}{r^2 P} \quad (17)$$

where

$$C_1 = \frac{\bar{T}\bar{\mu}\bar{Z}P_o Q_o}{KT_o(2\pi h)}$$

$$C_2 = F_B\left(\frac{P_o M}{RT_o}\right)^2\left(\frac{\bar{Z}\bar{T}R}{M}\right)\frac{Q_o|Q_o|}{(2\pi h)^2}$$

with the current sign convention where Q_o is positive for injection and negative for production.

Furthermore, Piplapure obtained a simplified analytical solution to Equation 17 which can be written as (45):

$$(P_e^2-P_w^2) = -\left[\frac{1424\bar{\mu}\bar{Z}\bar{T}Q_o}{hK}\ln\left(\frac{r_e}{r_w}\right) + \frac{3.1602\times10^{-12}\,F_B G\bar{Z}\bar{T}Q_o|Q_o|}{h^2}\left(\frac{1}{r_w}-\frac{1}{r_e}\right) + 2b(P_e-P_w)\right] \quad (18)$$

Examining a variety of experimental data (46,47), Flores compared the results obtained using Equation 18 to those obtained by the numerical integration of Equation 17 and concluded that the assumption in obtaining the simplified Equation 18 is not a severe one within the limits of his investigation.

Crafton (48) attempted to verify the Forchheimer equation for the radial flow systems. He applied Blick's method (49) to the radial flow systems by using a complete force-balance on the capillary-orifice model and proposed the following equation for steady plane radial gas flow through porous media:

$$-\frac{dP}{dr} = \left(\frac{C_L\,N_{Re}}{\pi r\phi d}\right)\mu q + \left(\frac{C_D}{2\sigma}+\frac{C_T}{\rho\pi r\phi}\right)\rho q|q| \quad (19)$$

A comparison of Equations 6 and 19 yields

$$F_A = \frac{C_L\,N_{Re}}{\pi\phi dr} \equiv \frac{1}{K} \quad (20)$$

and

$$F_B(r,\rho) = \frac{C_D}{2\sigma} \quad \text{"matrix term"} + \frac{C_T}{\rho\pi\phi r} \quad \text{"fluid-body term"} \quad (21)$$

where

C_L = laminar energy loss coefficient

C_T = turbulent energy loss coefficient

σ = distance between plates of orifice model

d = a characteristic hydraulic length

Dimensional inconsistency of Equation 21 (dimension of the matrix term as compared to the fluid-body term) indicates that $F_B(r,\rho)$ is not a parameter but it is a functional relationship. It should be noted that $F_B(r,\rho)$ becomes dimensionally consistent with F_B when the fluid-body term is neglected. Crafton suggested that the fluid-body term, which relates only the occurrences within the fluid, i.e. turbulence, should be represented by the volume including the radial increment σ. Hence, the following integration may be performed (50):

$$F_B(r,\rho) = \int_r^{r+\sigma} \left(\frac{C_D}{2\sigma} + \frac{C_T}{\pi\phi\rho r} \right) dr \qquad (22)$$

which will result in

$$F_B(r,\rho) = \frac{C_D}{2} + \frac{C_T}{\pi\phi\rho} \ln\left(\frac{r+\sigma}{r}\right) \qquad (23)$$

The radial increment distance, σ, was thought to represent the continuity correction (51).

Crafton (52) also derived an equation similar to Forchheimer's equation by using the Uberoi turbulent velocity decay solution which can be rewritten as

$$-\frac{dP}{dr} = \frac{1}{K}\mu q + \frac{F_C}{\mu}\rho q|q^{m-1}| \equiv \frac{1}{K}\mu q + B\rho q|q^{m-1}| \qquad (24)$$

and the comparison with Equation 6 yields

$$F_C = \mu q^{(2-m)} F_B \equiv \mu B \qquad (25)$$

where F_C is an empirical parameter which may be referred to as a "turbulent resistance coefficient", and m is the "derived turbulent intensity term".

Examination of Equations 23 and 25 indicate that the inertial coefficient F_B is not only a rock property but a fluid property as well.

It should be noted, however, that Crafton (53) in his original work used the symbol "B" to designate the quantity "ρF_B" when he applied Blick's method in the derivation, and "F_C/μ" when he applied the Uberoi turbulent velocity decay solution.

Furthermore, since the pressure gradient normally increases as the fluid viscosity increases, exactly opposite to that indicated by the second term of Equation 24, the validity of Crafton's analysis employing the Uberoi turbulent velocity decay solution is questionable. However, this invalidity was masked since Crafton included fluid viscosity in the definition of the inertial coefficient.

C. Correlations for b and F_B

The slippage coefficient b, for air, has been correlated with absolute permeability K by Heid et al. (54) as

$$b = 0.777\ K^{-0.39} \qquad (26)$$

where b is in international atmospheres and K is in millidarcies.

Cornell and Katz (55) proposed a simple relationship between the inertial resistance coefficient and permeability as

$$F_B = \frac{4.11 \times 10^{10}}{K^{4/3}} \qquad (27)$$

Kolada (56), assuming $F_B = f(K,\phi)$, proposed the following correlation by using available data in the literature (57,58,59,60):

$$F_B = \frac{7.56 \times 10^{8}}{\phi^{1.67}\ K^{1.12}} \qquad (28)$$

where K is in millidarcies, ϕ is a fraction and F_B is in ft^{-1}.

Considering the dimensional consistency as suggested earlier by Sadiq (61), Geertsma (62), in his recent work, suggested the following correlation:

$$F_B = \frac{0.005}{\phi^{5.5} K^{\frac{1}{2}}} \qquad (29)$$

where ϕ is a fraction, K is in ft^2 and F_B is in ft^{-1}.

CHAPTER III

THEORY

A. Visco-Inertial Flow Theory

The current theory describing the behavior of steady radial gas flow through porous media states that within the flow range normally experienced in gas reservoirs, including that immediately surrounding the wellbore, energy losses caused by turbulence can be safely ignored (63). It is currently believed that viscous and inertial forces simultaneously counteract the external force, with the inertial forces continuously gaining importance as the velocity increases.

A more general visco-inertial flow equation may be obtained to describe steady radial gas flow through porous media by including slippage effects together with the following assumptions:

1. The porous medium is homogeneous and isotropic with respect to permeability.
2. The system is of plane radial geometry and uniform thickness.
3. Flow is horizontal, i.e. gravitational forces are neglected.
4. The formation is saturated with a single homogeneous fluid, i.e. gas, with constant composition.
5. The continuity equation is valid, i.e.

$$\frac{1}{r}\frac{\partial}{\partial r}[r(\rho q)] = -\frac{\partial(\rho\phi)}{\partial t} \tag{30}$$

which reduces to

$$\rho Q = \rho_o Q_o \quad \text{for steady-state flow} \tag{31}$$

6. Changes in gas properties, i.e. μ, Z and ρ, can be evaluated at the arithmetic mean temperature and pressure.

7. Slippage effects can be taken into account by means of the equation

$$K_a = K\left(1 + \frac{b}{P}\right) \tag{32}$$

8. The equation of state for real gases is given by

$$\rho = \frac{PM}{ZRT}, \quad \text{and} \quad \rho_o = \frac{P_o M}{RT_o} \tag{33}$$

9. Forchheimer's equation is valid, i.e. Darcy and Non-Darcy flows are considered, as follows:

$$-\frac{dP}{dr} = \frac{\mu}{K_a} q + F_B \rho q|q| \tag{34}$$

Combining Equations 31 through 34 under the stated assumptions, Piplapure (64) developed the following non-linear differential equation which has as yet no known analytical solution:

$$-P\frac{dP}{dr} = \frac{C_1}{\left(1 + \frac{b}{P}\right)r} + \frac{C_2}{r^2} \tag{35}$$

and

$$Q_o = q_o(2\pi hr)$$

General steady plane-radial visco-inertial flow models can now be developed.

1. Differential Model

Equation 35 can be rewritten, with the customary field units, to yield what may be called the "differential model" flow equation

$$\frac{dP}{dr} = -\left[\frac{D_1}{K(P+b)r} + \frac{F_B D_2}{Pr^2}\right] \tag{36}$$

where

$$D_1 = \frac{712\bar{\mu}\bar{Z}\bar{T}Q_o}{h}$$

$$D_2 = \frac{1.5801 \times 10^{-12}\, G\bar{T}\bar{Z}Q_o|Q_o|}{h^2}$$

This represents the rigorous model for steady radial gas flow through porous media with parameters K, b and F_B.

2. Integrated Model

Equation 35 may also be written as

$$-P\frac{dP}{dr} = \frac{C_1 r + C_2\left(1 + \frac{b}{P}\right)}{\left(1 + \frac{b}{P}\right)r^2} \tag{37}$$

If the term $(\frac{b}{P})$ is sufficiently small, i.e. $C_2 >> C_2(\frac{b}{P})$, the term $C_2(\frac{b}{P})$ in the numerator may be neglected which reduces Equation 31 to

$$-P\frac{dP}{dr} = \frac{C_1 r + C_2}{\left(1 + \frac{b}{P}\right)r^2} \tag{38}$$

This equation is now separable and integration yields

$$(P_e^2-P_w^2) = -\left[\frac{1424\bar{\mu}\bar{Z}\bar{T}Q_o}{hK}\ln\left(\frac{r_e}{r_w}\right)\right. \quad \text{viscous-term}$$

$$\left. + \frac{3.1602\times10^{-12}\,F_B G\bar{T}\bar{Z}Q_o|Q_o|}{h^2}\left(\frac{1}{r_w}-\frac{1}{r_e}\right)\right] \quad \text{inertial-term}$$

$$- 2b(P_e-P_w) \quad \text{slippage-term} \qquad (39)$$

This represents a simplified model with parameters K, b, and F_B (65,66).

B. Turbulent Flow Theory

In his work, Crafton (67) assumes that the structure of turbulence is locally isotropic and inhomogeneous and may be described by the Uberoi turbulent velocity decay solution (68). He proposed Equation 24 to describe turbulent flow through porous media. Since the derivation of this equation was not clear, an attempt was made to redrive it. This derivation is presented in Appendix A. Since its validity is questionable, further usage of Equations 24 and/or A-11 was avoided.

The customary field units in these mathematical models are defined as follows:

Q_o	= MSCF/D	h	= ft.
$\bar{\mu}$	= cp	r_e, r_w	= ft.
K	= mds	b	= psia

F_B = ft^{-1} m = dimensionless

$F_C = \frac{(lbs)(day)^{m-3}}{(ft)^m}$ G = gas gravity, with $G_{air} = 1.0$

T_o = 520°R P_o = 14.697 psia

C. Concept of Rock Confinement Pressure

The experimentally measured values of interrelated rock properties, i.e. rock compressibility, porosity, permeability, hence the slippage and inertial coefficients, are dependent upon the rock confinement pressure used.

By definition, the confinement pressure in the crust of the earth is the lithostatic pressure on the reservoir rock resulting from the load of overlying rocks. In an experimental work, the confinement pressure is a hydrostatic pressure on the rock sample, generally produced by liquids. Confinement pressure means equal, allsided pressure.

1. Net Confinement Pressure

In an experimental work, the same stress state of rock can be maintained provided that the applied hydrostatic confinement pressure stays constant. Furthermore, a net hydrostatic confinement pressure on the rock sample can be maintained constant provided the mean flowing fluid pressure does not change. However, it is very hard, if not impossible, to maintain a constant mean flowing pressure

and obtain the pertinent data at the same time. Therefore, one may define a net confinement pressure which remains constant, even if the mean flowing fluid pressure changes, as follows:

$$P_{nc} = P_{hc} - \bar{P} \tag{40}$$

where

P_{nc} = net rock confinement pressure, psig

P_{hc} = hydrostatic rock confining pressure, psig

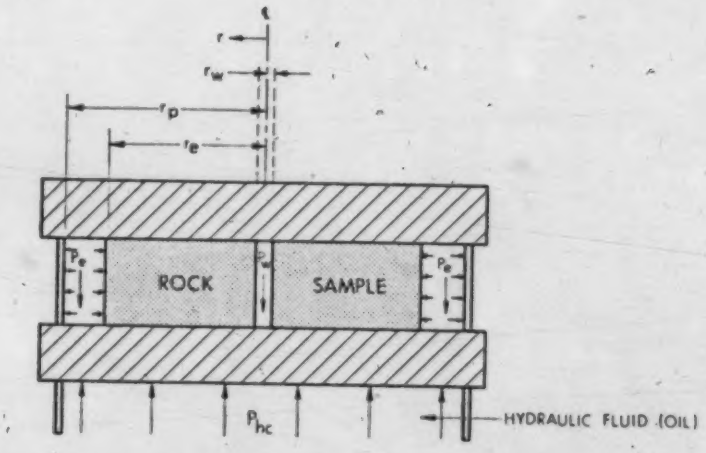
$\bar{P}$ = mean flowing fluid pressure in the rock sample, psig

It should be noted that Equation 40 as used by Piplapure (74), assumes that the rock sample completely fills the particular flow cell used in the experimental work.

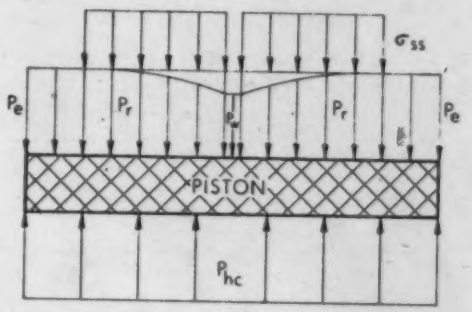
2. Proposed Net Confinement Pressure

Determination of net confinement pressure, i.e. hydrostatic pressure, on the rock sample used in an experimental work requires a complete force balance on the sample holder. A cross section of the sample holder and the free body diagram of the forces on the hydraulic piston is shown in Figure 1.

A force balance, in the direction perpendicular to that of fluid flow, can be written as



(a) A section of the Radial Flow Sample Holder



(b) Force - balance on the hydraulic piston

Figure 1. RADIAL FLOW CELL AND FREE BODY DIAGRAM OF FORCES ON THE PISTON.

$$P_{hc}(\pi r_p^2) = P_e(\pi r_p^2 - \pi r_e^2) + P_w(\pi r_w^2) + \sigma_{ss}(\pi r_e^2 - \pi r_w^2) + \int_{r_w}^{r_e} P(2\pi r)dr \quad (41)$$

where r_p is the radius of the hydraulic piston. This equation can be solved for the "structural support pressure, σ_{ss}" (75) which is the net confinement pressure on the rock sample, as follows:

$$P_{nc} = \left(\frac{r_p^2}{r_e^2 - r_w^2}\right) P_{hc} - \left[\left(\frac{r_p^2 - r_e^2}{r_e^2 - r_w^2}\right) P_e + \left(\frac{r_w^2}{r_e^2 - r_w^2}\right) P_w + \frac{2}{(r_e^2 - r_w^2)} \int_{r_w}^{r_e} (P \cdot r)dr \right] \quad (42)$$

Using Brant's idea (76), which is also supported by Fatt (77), that only 85% of the internal fluid pressure in the pores reacts against the external pressure, i.e. the hydraulic pressure on the rock sample, Equation 42 can be reduced to

$$P_{nc} = \left(\frac{r_p^2}{r_e^2 - r_w^2}\right) P_{hc} - \left[\left(\frac{r_p^2 - r_e^2}{r_e^2 - r_w^2}\right) P_e + \left(\frac{r_w^2}{r_e^2 - r_w^2}\right) P_w + \left(\frac{1.70}{r_e^2 - r_w^2}\right) A(r) \right] \quad (43)$$

where

$$A(r) = \int_{r_w}^{r_e} (P \cdot r)dr \quad (44)$$

Assuming that Darcy's Law may be used to describe the fluid

pressure distribution, then the pressure at any radius r can be written as

$$P^2 = P_e^2 + \frac{(P_e^2-P_w^2)}{ln\left(\frac{r_e}{r_w}\right)} ln\left(\frac{r}{r_e}\right) \tag{45}$$

or

$$P = \left[P_e^2 + \frac{(P_e^2-P_w^2)}{ln\left(\frac{r_e}{r_w}\right)} ln\left(\frac{r}{r_e}\right)\right]^{\frac{1}{2}} \tag{46}$$

Consequently, Equation 50 can be rewritten as

$$A(r) = \int_{r_w}^{r_e} \left[P_e^2 + \frac{(P_e^2-P_w^2)}{ln\left(\frac{r_e}{r_w}\right)} ln\left(\frac{r}{r_e}\right)\right]^{\frac{1}{2}} \cdot r \cdot dr \tag{47}$$

The integral term can not be evaluated analytically but it can be integrated numerically by employing the Trapezoidal rule.

CHAPTER IV

PARAMETER ESTIMATION TECHNIQUES

In this work, the integrated model of the visco-inertial flow theory is used for parameter estimation so that both numerical and graphical estimation techniques can be presented. Equation 39, considering production only, can be rewritten as

$$P_e^2-P_w^2 = \frac{1424\bar{\mu}\bar{Z}\bar{T}Q_o ln(r_e/r_w)}{hK} - 2b(P_e-P_w) + \frac{3.1602 \times 10^{-12} F_B G \bar{T}\bar{Z}Q_o^2}{h^2}\left(\frac{1}{r_w}-\frac{1}{r_e}\right) \quad (48)$$

with parameters K, b and F_B to be estimated.

A. Numerical Parameter Estimation Technique: The Gauss-Newton Method

Parameters for the integrated model, Equation 54, can be estimated by using quasilinearization (78,79) or by applying the Marquart algorithm for non-linear least squares parameter estimation (80).

The Gauss-Newton method is used in this work to estimate the unknown parameters K, b and F_B. The Marquart algorithm is an extension of the Gauss-Newton method to assure convergence with relatively poor starting guesses for the unknown parameters, by introducing a factor λ. It can be shown that when λ is equal to zero, the Marquart algorithm reduces to the Gauss-Newton method (81,82).

Initial guesses for the unknown parameters can be estimated in the following manner:

1. Permeability, K, can be approximated from a single point flow test utilizing the lowest flow rate observation.
2. Knowing the value of K, one can estimate the slippage coefficient, b, by means of Equation 26 which can be rewritten as

$$b = 11.42\ K^{-0.39} \tag{49}$$

where K is in millidarcies and b is in psia.

3. When K and porosity, ϕ, are known, the inertial resistance coefficient, F_B, can be approximated by using Equation 28.

The Gauss-Newton method converges to parameters faster than the quasilinearization method. It also converges faster than the Marquart algorithm provided that good initial estimates for the unknown parameters can be obtained.

At this point it is appropriate to state that any numerical technique, i.e. the Gauss-Newton method, furnishes the estimates for the unknown parameters using the data, irrespective of their quality. Hence care must be taken in its usage.

Application of the Gauss-Newton method to Equation 48, together with the appropriate computer programs, is presented in Part I of Appendix B.

B. Graphical Parameter Estimation Techniques

It is important to apply some kind of graphical technique in parameter estimation in order to

1. Check for anomalous experimental data, and

2. Provide a solution when one does not have access to a digital computer as required by any numerical technique.

Application of graphical techniques requires a proper delineation of the viscous and the inertial flow regions. Hence, the accuracy of the estimated parameters depends upon the quality of the data splitting technique used.

1. A Graphical Estimation Technique

This technique requires two plots to estimate the unknown parameters K, b and F_B which can be presented in the following fashion:

(a) The Klinkenberg Plot

Assuming the flow is viscous, i.e. $F_B = 0$, Equation 48 can be rearranged to yield

$$K_a = \frac{1424\bar{\mu}\bar{Z}\bar{T}\ln(r_e/r_w)}{h(P_e^2-P_w^2)} = C_A \frac{\bar{\mu}\bar{Z}\bar{T}}{(P_e^2-P_w^2)} \qquad (50)$$

where

$$C_A = \frac{1424\ln(r_e/r_w)}{h}$$

and K_a is also given by Equation 2. A plot of K_a versus $(1/\bar{P})$, known as the "Klinkenberg plot", yields a straight line from which K and b may be determined.

It should be noted that the Klinkenberg plot will furnish good estimates of K and b provided the data used are in the viscous region only.

(b) The Modified Visco-Inertial Plot

Rearrangement of Equation 48 yields

$$\frac{P_e^2-P_w^2 + 2b(P_e-P_w)}{\bar{\mu}\bar{Z}\bar{T}Q_o} = \frac{C_A}{K} + C_B F_B \left(\frac{Q_o}{\bar{\mu}}\right) \qquad (51)$$

where

$$C_B = \frac{3.1602 \times 10^{-12}\, G}{h^2}\left(\frac{1}{r_w} - \frac{1}{r_e}\right)$$

Hence, a plot of the left hand side of Equation 51 versus $(Q_o/\bar{\mu})$ will furnish a straight line from which the values of K and F_B may be determined provided that the slippage coefficient, b, is already known and the data used are in the region where the inertial effects are also significant, i.e. the visco-inertial region. This plot is commonly referred to as the "modified visco-inertial plot".

2. A Proposed Semi-Graphical Parameter Estimation Technique

There are two fundamental problems in this technique and these may be expressed in the following manner:

1. The first problem is to determine the extent of the experimental data in hand, i.e. points in viscous and/or in inertial region(s). This may be accomplished by using all the data points in the construction of the Klinkenberg plot, as given by Equation 2. The straight line and concave-downward curved portions of which represent the viscous and the inertial regions, respectively. Thus, the Klinkenberg plot will indicate any anomalous data points, and will indicate qualitatively,

the type(s) of flow region(s) present.

2. The second problem is to delineate the viscous and inertial flow data which, in turn, are to be used to obtain the simultaneous linear least squares estimates for K_v and b, and K_i and F_B using Equations 2 and 51, respectively. However, the appropriate values for K_v, b, K_i and F_B must be determined using the following proposed data discrimination methods:

Method 1. Find the best fit for the Klinkenberg plot, given by Equations 2 and 50, i.e. determine the maximum coefficient of correlation which may be written as

$$R_{vi} = \frac{n \sum_{i=1}^{n} K_{ai} \left(\frac{1}{\bar{P}_i}\right) - \left(\sum_{i=1}^{n} K_{ai}\right)\left(\sum_{i=1}^{n} \frac{1}{\bar{P}_i}\right)}{\left\{\left[n \sum_{i=1}^{n}\left(\frac{1}{\bar{P}_i}\right)^2 - \left(\sum_{i=1}^{n} \frac{1}{\bar{P}_i}\right)^2\right]\left[n \sum_{i=1}^{n} K_{ai}^2 - \left(\sum_{i=1}^{n} K_{ai}\right)^2\right]\right\}^{\frac{1}{2}}} \quad (52)$$

where R_{vi} is the coefficient of correlation, and can range between -1 to +1, indicating the perfect inverse and perfect direct relationship respectively, with n number of data points assumed to be in the viscous region.

Method 2. Split the total number of data points such that the difference between the estimated permeability values, K_v and K_i, is a minimum. The relative error between the estimated permeability values, ε_i, is given by

$$\varepsilon_i = \frac{(K_v)_i - (K_i)_j}{(K_v)_i} \tag{53}$$

where i and j are the number of data points in viscous and inertial regions, respectively.

Method 3. Estimate the Reynolds number, N_{Re}, which is a measure of the ratio of inertial effects to viscous effects as defined by Hubbert (83). At this point it is appropriate to redefine the Reynolds number including slippage effects, as,

$$N_{Re} = \frac{\text{inertial effects}}{\text{viscous and slippage effects}} \tag{54}$$

which, from Equation 49, can be written as

$$N_{Re} = \frac{C_B F_B \bar{\bar{Z}}\bar{\bar{T}} Q_o^2}{C_A \bar{\bar{\mu}}\bar{\bar{Z}}\bar{\bar{T}} Q_o / K_v - 2b(P_e - P_w)} \tag{55}$$

At Reynolds numbers less than 0.01, the flow is commonly considered to be viscous and inertial effects can be neglected.

Consequently, the proposed semi-graphical parameter estimation technique requires a Klinkenberg plot and a simultaneous linear least squares fit to both Equations 2 and 51 in order to split the total experimental data and estimate the unknown parameters K, b and F_B.

The FORTRAN computer programs necessary to perform this task are presented in Part II of Appendix B.

CHAPTER V

EXPERIMENTATION

A. Experimental Apparatus

The experimental apparatus used was essentially the same as that employed by Piplapure (84). It consists of a porosimeter, a flow test apparatus and a sample holder, i.e. flow cell, as shown in Figures 2, 3 and 4 respectively.

B. Experimental Procedures

1. Preparation of Rock Samples

Five rock samples, each of about 12" diameter and 1" thickness, were used in this study. Three of these were the same type of Indiana Limestone samples that were used by Piplapure (85). The other two were Blairmore Sandstone samples which were cut in the Department of Mineral Engineering Machine Shop with a specially designed saw.

Uniaxial compression tests were necessary in order to determine the maximum confinement pressure that could be applied to the rock sample to assure a safe test without breaking the sample. This was achieved by testing six cylindrical core samples, having lengths at least twice their diameters, in the Rock Mechanics Laboratory of the Mineral Engineering Department (86).

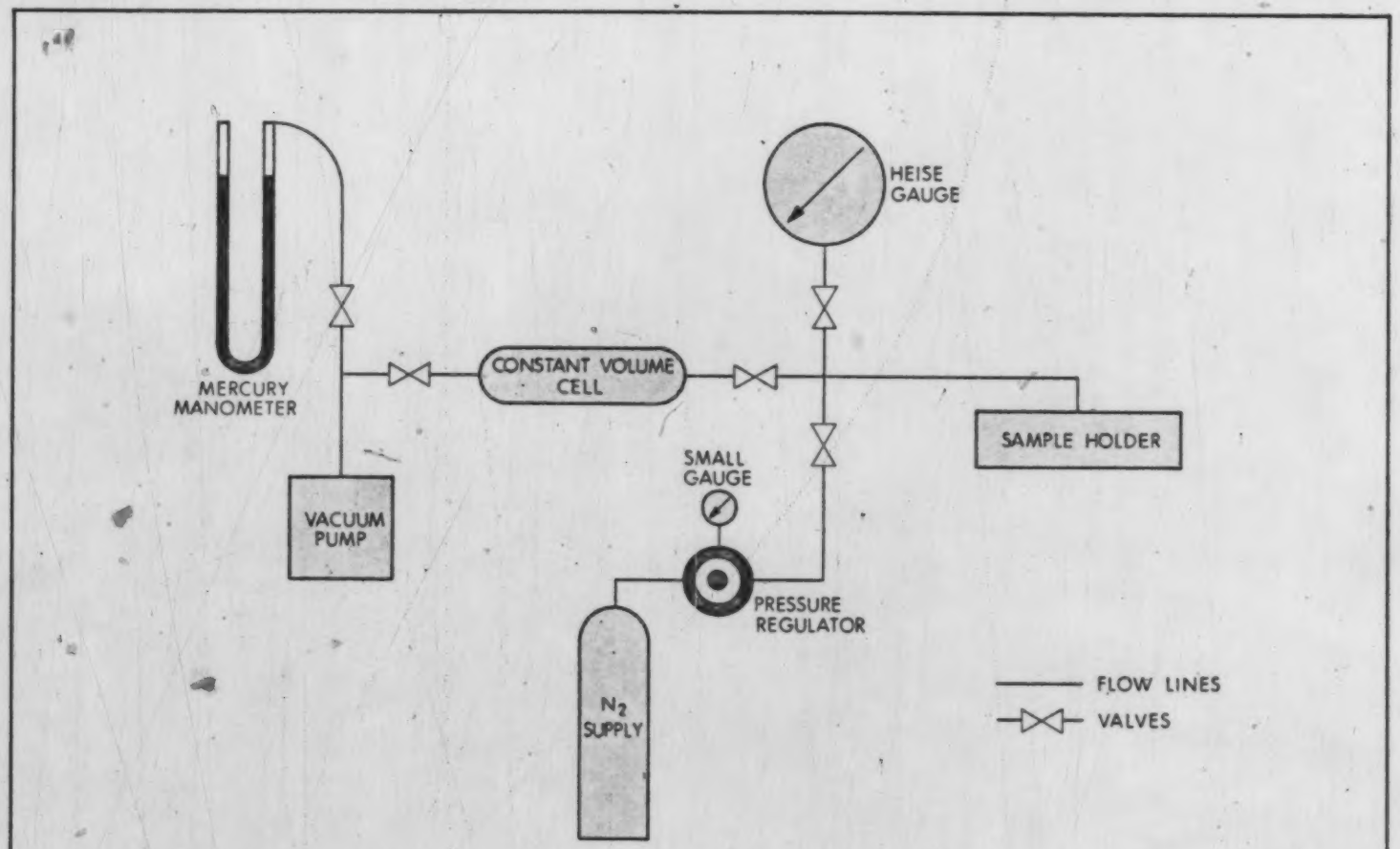


Figure 2. A FLOW SYSTEM LAYOUT FOR THE POROSIMETER

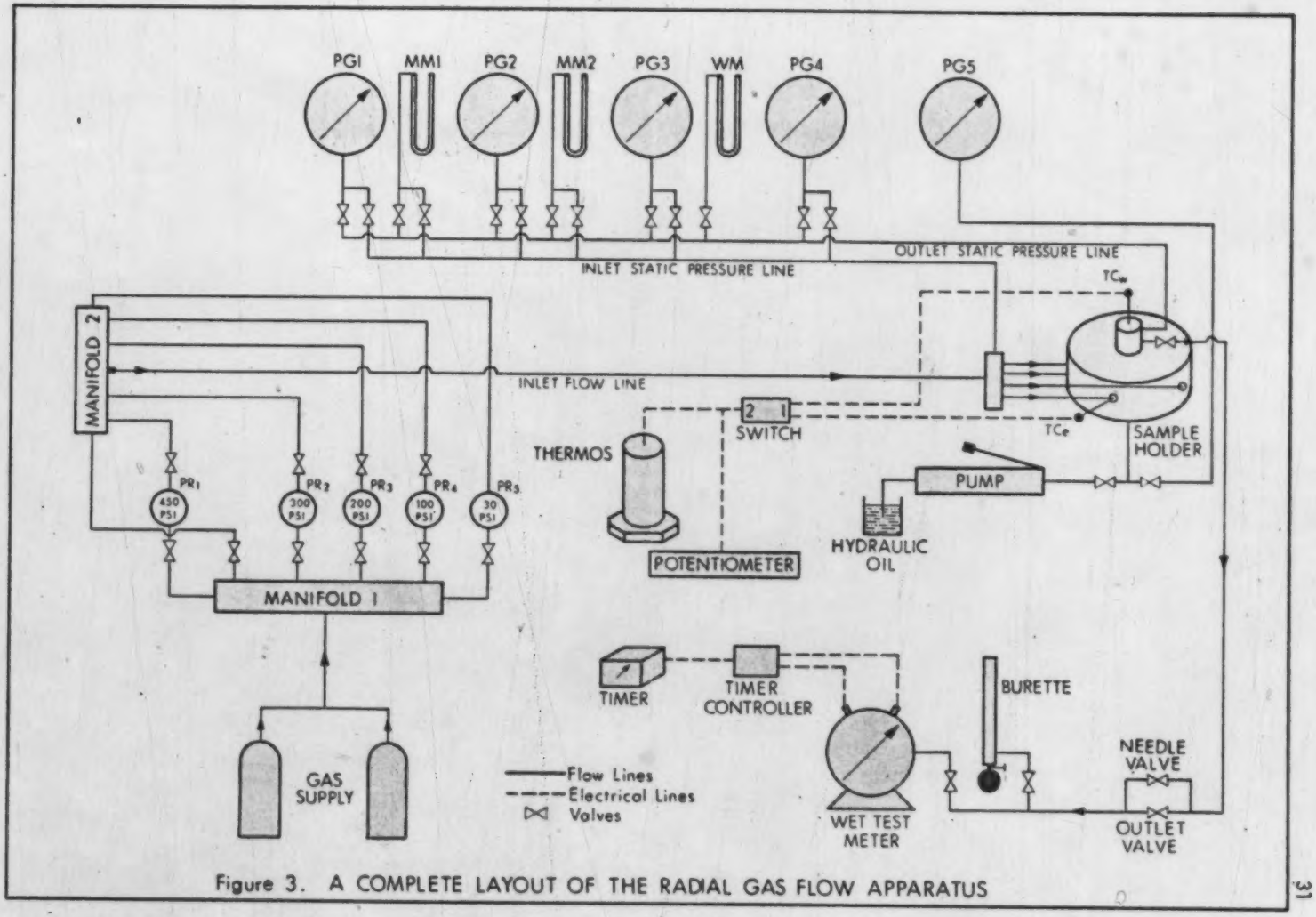


Figure 3. A COMPLETE LAYOUT OF THE RADIAL GAS FLOW APPARATUS

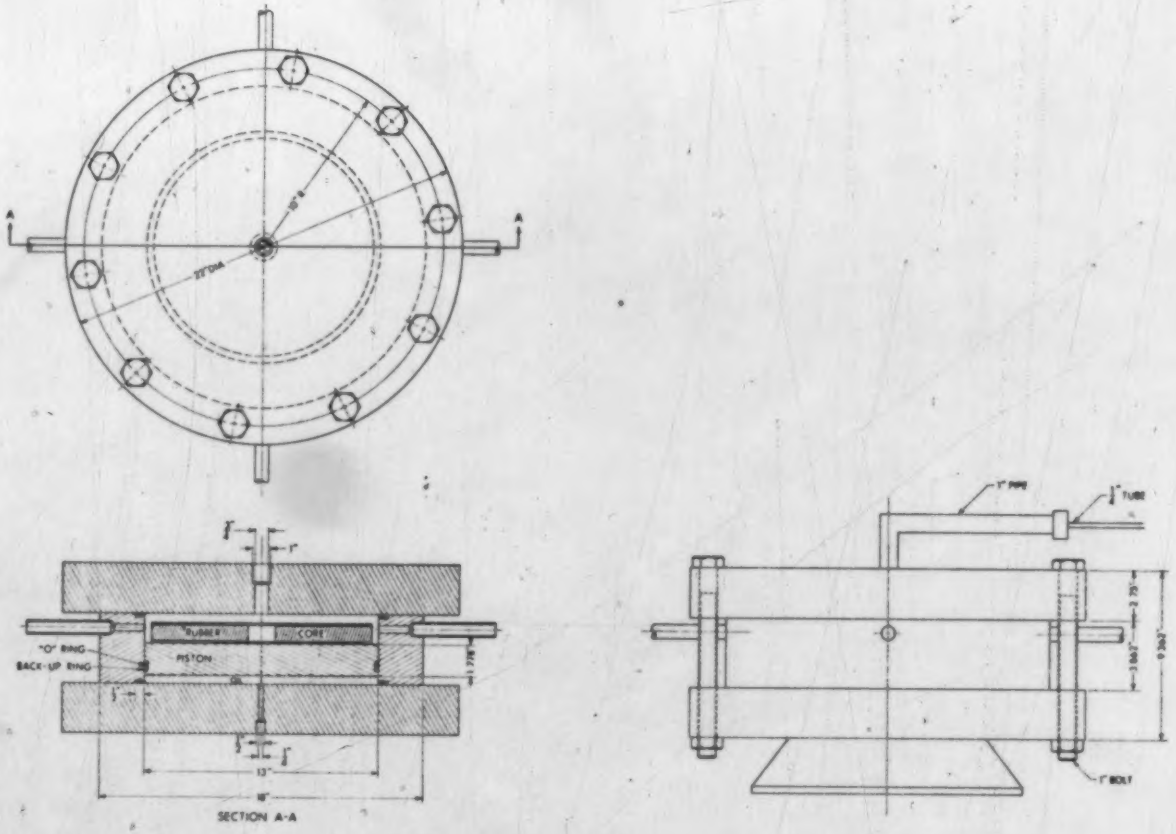


Figure 4. RADIAL FLOW CELL

2. Operational Procedure for Radial Gas Flow Apparatus

Using the radial gas flow apparatus in conjunction with the porosimeter, the experimental data were obtained in the following manner:

1. In order to determine the effective porosity, the bulk volume of each of the samples was evaluated by measuring the thickness and the inner and outer diameters. The effective grain volume of each sample was determined using the gas expansion porosimeter, as shown in Figure 2, which was previously calibrated with solid aluminium blanks of known dimensions.
2. Devices such as the pressure gauges, wet-test meter, timer, and potentiometer were appropriately calibrated and connected to the apparatus assembly, as shown in Figure 3, prior to the flow tests.
3. All the fittings and connections were pressure tested to ensure a leak-proof assembly.
4. The rock sample was mounted inside the flow cell and subjected to the desired confinement pressure measured by pressure gauge, PG_4.
5. Gas from the supply cylinder was allowed to flow through the appropriate pressure regulator, PR, at the desired upstream pressure.
6. Gas was allowed to flow for a sufficient time to ensure stabilized conditions. This was checked by successive flow rate measurements.

7. A soap film burette and a wet-test meter, both connected to the gas outlet, were used to measure low and high flow rates, respectively.
8. Inlet and outlet pressures were measured with the appropriate pressure gauge (PG_1, PG_2 or PG_3), or with a mercury manometer (MM_1 or MM_2) provided that only one device at a time was used for either inlet or outlet pressure measurement. Water manometer, WM, was used for very low outlet pressure measurements.
9. Inlet and outlet temperatures were measured with Copper-Constantan Thermocouples TC_e and TC_w, respectively.
10. Barometric pressure was also recorded during every experimental run at appropriate time intervals.

3. Flow Test Methods

Generally, the flow tests were conducted under a constant net confinement pressure in order to obtain reliable measurements. For a constant value of the net confinement pressure, the hydrostatic confinement pressure, P_{hc}, as may be seen from Equation 43, is no longer constant but depends on the values of upstream and downstream pressures used in the particular flow test. Hence, there exists a distribution of hydrostatic confinement pressure for every predefined net confinement pressure depending upon the way in which the flow test is conducted.

In this investigation two flow-test methods were used to conduct experimental runs and may be referred to as

1. Case I

   Tests were conducted under a predetermined net rock confinement pressure by varying the upstream pressure at a relatively constant downstream pressure during each run.

2. Case II

   Tests were conducted under a predetermined net rock confinement pressure at a constant upstream pressure by varying the downstream pressure during each run.

The required hydrostatic rock confinement pressure for these test methods may be calculated using Equation 43 in the form

$$P_{hc} = \left[P_{nc} + \left(\frac{r_p^2 - r_e^2}{r_e^2 - r_w^2} \right) P_e + \left(\frac{r_w^2}{r_e^2 - r_w^2} \right) P_w + \left(\frac{1.70}{r_e^2 - r_w^2} \right) A(r) \right] \left(\frac{r_e^2 - r_w^2}{r_p^2} \right) \tag{56}$$

where A(r) is given by Equation 47. Furthermore, Equation 40 may be rewritten, in terms of P_{hc}, as

$$P_{hc} = P_{nc} + \frac{1}{2}(P_e + P_w) \tag{57}$$

These equations were used to obtain the appropriate type curves utilized in this study.

CHAPTER VI

TREATMENT OF DATA

Observed pressure, temperature and flow rate readings were corrected and reduced to standard conditions by applying the appropriate calibration and conversion factors. Volumetric flow rates were converted to the reference conditions of 1 international atmosphere, i.e. 14.697 psia, and 519.7°R.

A. Evaluation of Fluid Properties

In determining fluid properties, the values of compressibility and viscosity were evaluated at the arithmetic mean temperature and pressure.

1. Gas Compressibility Factor

Compressibility factors for nitrogen and argon were evaluated by a five-point Lagnangian interpolation of the curve fitted data of Hilsenrath et al. (87) at temperatures of 504, 522, 540, 558, and 576°R.

Compressibility factors for methane were determined by using the Beattie-Bridgeman equation of state. The Beattie-Bridgeman equation may be written as

$$\bar{P} = \frac{R\bar{T}}{V} + \frac{\beta}{V^2} + \frac{\gamma}{V^3} + \frac{\delta}{V^4} \qquad (58)$$

where $\bar{P}$ is the arithmetic mean pressure, V is the molal volume, $\bar{T}$ is the arithmetic mean temperature, β, γ and δ are temperature-dependent

characteristics of the gas, and R is the universal gas constant in compatible units. The parameters are defined by the relationships

$$\beta = B_o R\bar{T} - A_o - \frac{cR}{\bar{T}^2} \tag{59}$$

$$\gamma = -bB_o R\bar{T} + aA_o - \frac{cB_o R}{\bar{T}^2} \tag{60}$$

$$\delta = \frac{RbcB_o}{\bar{T}^2} \tag{61}$$

where A_o, B_o, a, b, and c are tabulated constants determined empirically from experimental data and which are different for each gas. For methane, the values of 2.2769, 0.05587, 0.01855, -0.01587 and 12.8300 were used for A_o, B_o, a, b, and c, respectively (88).

Equation 58 is explicit in pressure $\bar{P}$ but implicit in temperature $\bar{T}$ and volume V. Hence, some iterative root finding procedure is required to find the volume which corresponds to given values of $\bar{P}$ and $\bar{T}$. Therefore, Newton's method was used to solve Equation 58 for molal volume which, in turn, was used to calculate the compressibility factor using

$$\bar{Z} = \frac{\bar{P}V}{R\bar{T}} \tag{62}$$

where $\bar{P}$ is in atmospheres, $\bar{T}$ is in °R, V is in liters/gm-mole and R is 0.08205 (cc atm)/(gm-mole °K).

2. Gas Viscosity

Viscosities for both nitrogen and argon were obtained using the relationships suggested by Kestin and Wang (89) as

$$\bar{\mu} = 1.778 \times 10^{-4}[1 + 8.958 \times 10^{-4}(\bar{P}-1) + 6.120 \times 10^{-7}(\bar{P}-1)^2 + 3.997 \times 10^{-8}(\bar{P}-1)^3] + 4.55 \times 10^{-7}(\bar{T}-25) \tag{63}$$

and

$$\bar{\mu} = 2.262 \times 10^{-4} [1 + 8.945 \times 10^{-4}(\bar{P}-1) + 4.930 \times 10^{-6}(\bar{P}-1)^2 + 7.200 \times 10^{-8}(\bar{P}-1)^3] + 6.37 \times 10^{-7}(\bar{T}-25) \quad (64)$$

respectively, where $\bar{\mu}$ is in poise, $\bar{P}$ is in international atmospheres and $\bar{T}$ is in °C.

The viscosity of methane, at an arithmetic mean temperature and pressure, was obtained using the correlation of Coremans and Beenakker (90) which is written as

$$\bar{\mu} = \bar{\mu}_o [1 + (0.55\rho^* + 0.96\rho^{*2} + 0.61\rho^{*3}) T^{* -0.59}] \quad (65)$$

where

$\bar{\mu}_o = \frac{0.002669\sqrt{M\bar{T}}}{\sigma^{*2}\Omega_v}$, low-pressure viscosity at the same temperature, cps

M = molecular weight of the gas

σ^* = hard-sphere diameter, angstroms

$\Omega_v^{-1} = 0.697(1 + 0.323 \ln T^*)$

$\rho^* = \bar{\rho}b_o$, dimensionless

b_o = hard-sphere volume, cm^3/gm-moles

$T^* = k\bar{T}/\varepsilon_o$, where ε_o/k is the Lennard-Jones potential parameter

k = Boltzman constant

ε_o = maximum energy of molecular attraction, ergs

and $\bar{\rho}$ is the arithmetic mean density, in gm-moles/cm^3, which may be evaluated by

$$\bar{\rho} = \frac{\bar{P}}{\bar{Z}R\bar{T}} \quad (66)$$

The values of constants σ^*, ε_o/K, and b_o used were 148.6, 3.758 and 66.98, respectively.

Subroutine FLUID was written to calculate these fluid properties and is presented in Part I of Appendix B.

B. Evaluation of Results

Using the estimated values of F_B for the runs with different gases and under the same net confinement pressure, the possibility of F_B being a property of fluid as well as the rock was examined.

The effect of variations in rock stress on absolute permeability K, has been studied by Fatt (91). Consequently, it is known that as rock stress, i.e. confinement pressure, increases, absolute permeability decreases and this effect is more severe in low permeability rocks than in high permeability rocks. This particular effect can be illustrated by defining a dimensionless factor

$$f_K = \frac{\text{K at any rock confinement pressure}}{\text{K at the lowest possible rock confinement pressure}} \quad (67)$$

and plotting this factor versus the net rock confinement pressure.

Porosity is also affected by an increase in rock stress (92), decreasing as rock stress increases.

Using the values of b and F_B for different runs conducted under several net rock confinement pressures, the effect of variations in rock stress on these parameters was also examined.

CHAPTER VII

RESULTS AND DISCUSSION

A. Net Rock Confinement Pressure

Equations 56 and 57 were used to determine the hydrostatic confinement pressure distributions for every predefined net rock confinement pressure and flow test method. In this regard the integral term $\bar{A}(r)$, as given by Equation 47 and employed in Equation 56, was evaluated by means of the Trapezoidal rule, with

step size, $\Delta r = 0.25"$

$r_e = 6.00"$

$r_w = 0.125"$

and $r_p = 6.50"$

As a result of these evaluations, the following family of type curves was obtained for each flow test method:

1. Case I: *varying upstream pressure P_e, and constant downstream pressure P_w*

   For this case, the algebraic expression for the family of type curves appropriate for calculating hydrostatic rock confinement pressure distributions was found to be

   $$P_{hc} = (0.86\ P_{nc} + 0.82\ P_e) \pm 10, \quad 0 \leq P_w \leq 300 \qquad (68)$$

   A comparison of hydrostatic pressure distributions, for $P_{nc} = 600$ psig, as calculated by this equation and Equation 57 is shown in Figure 5.

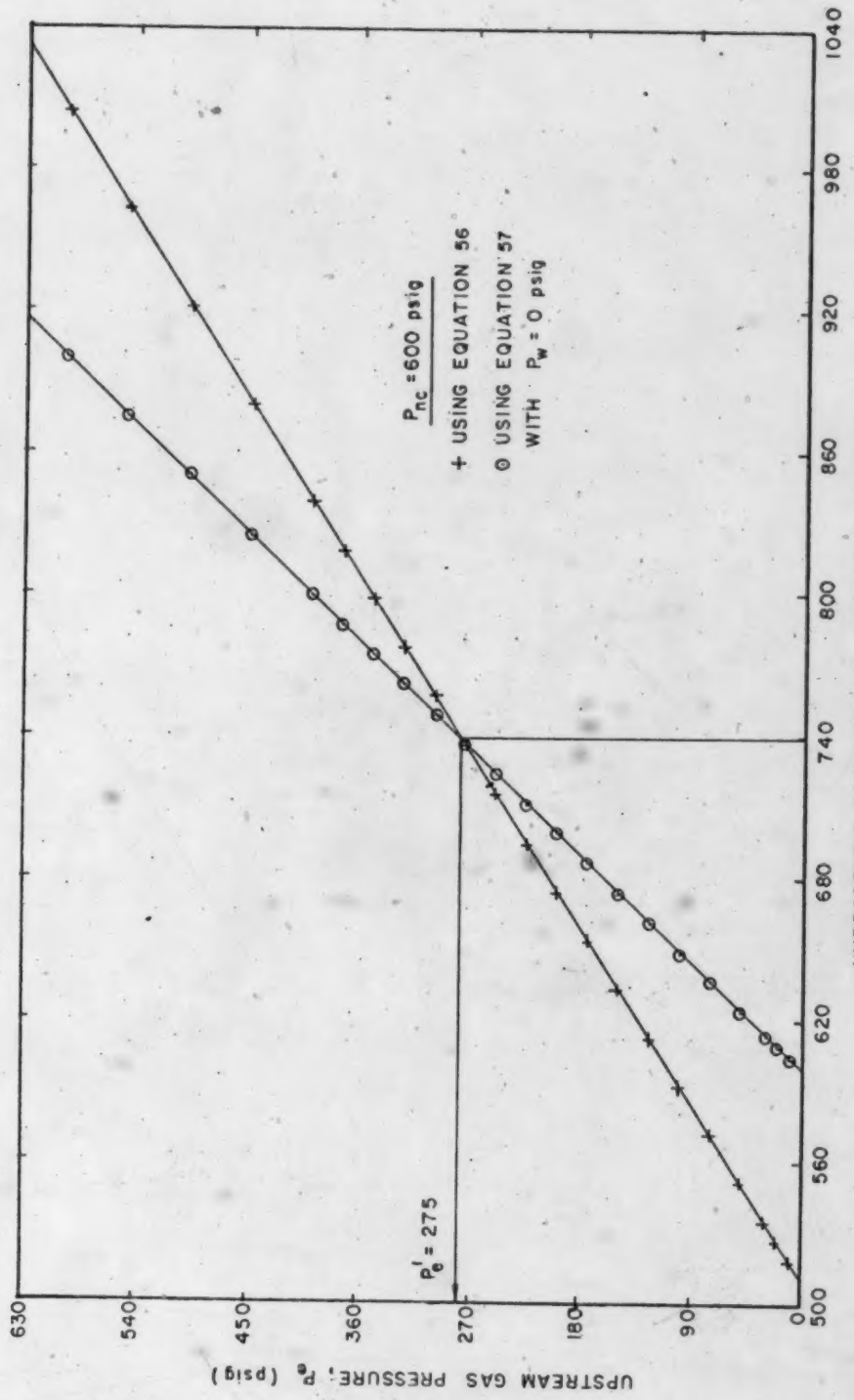


FIGURE 5 P_e VERSUS P_{hc} TYPE CURVES FOR FLOW TEST METHOD CASE - I

2. Case II: *varying downstream pressure P_w, and constant upstream pressure P_e*

For this case, the family of type curves expressing the hydrostatic rock confinement pressure distributions may be approximated by the expression

$$P_{hc} = (0.86\ P_{nc} + 250) \pm 5, \quad P_e = 300 \text{ psig}$$
$$0 \leq P_w < 300 \qquad (69)$$

A comparison of hydrostatic pressure distributions, for P_{nc} = 600 psig and P_e = 300 psig, as calculated by Equations 56 and 57 is presented in Figure 6. However, the approximate solution for this case, Equation 69, gives a constant hydrostatic rock confinement pressure instead of pressure distribution, i.e. P_{hc} = 766 ± 5 psig.

Comparison of Equations 57 and 68 both yield the same hydrostatic confinement pressure when

$$P_e = P'_e = \left(\frac{7}{16} P_{nc} + \frac{25}{16} P_w\right) \pm 10 \qquad (70)$$

and for the specific case presented in Figure 5, P'_e = 275 psig. For P_e less than P'_e the rock is compressed more by the application of Equation 57 than the application of Equation 68 and when P_e is greater than P'_e, the opposite is true.

Similarly a comparison of Equation 57 and 69 yields the same hydrostatic confinement pressure when

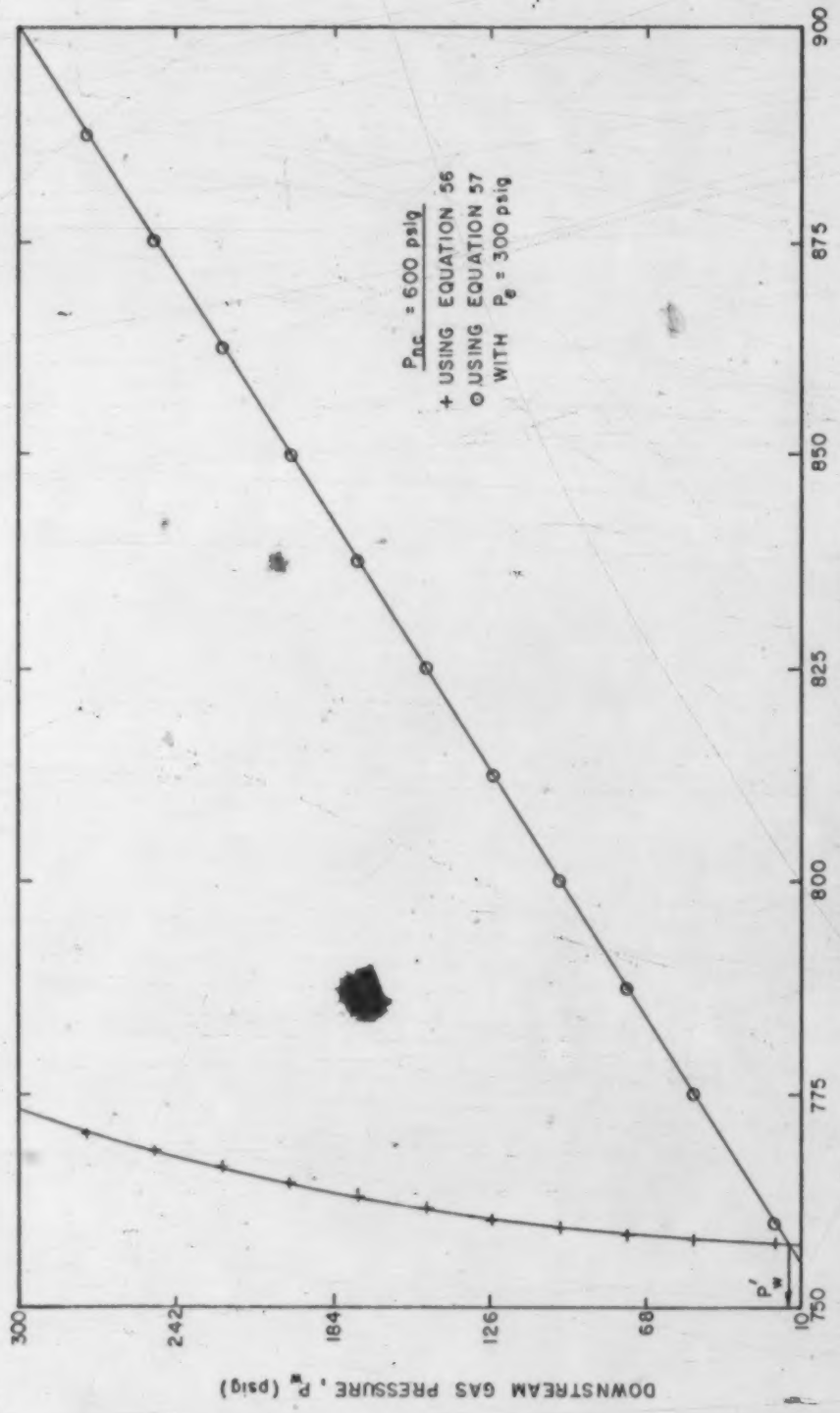


FIGURE 6 P_w VERSUS P_{hc} TYPE CURVES FOR FLOW TEST METHOD CASE - I I

$$P_w = P'_w = \left(200 - \frac{7}{25} P_{nc}\right) \pm 10 \qquad (71)$$

and for the specific case, i.e. P_{nc} = 600 psig, $P'_w = 32 \pm 10$. However, this value is about 15 psig, as shown in Figure 6, when Equations 56 (instead of 69) and 57 are compared.

Thus, for essentially all values of P_w, the rock is compressed more by the application of Equation 57 instead of Equations 56 and/or 68, 69.

Consequently, these findings suggest that the radial gas flow data obtained by Piplapure (93) were taken under a state of higher compression of the rock samples since Equation 57 was used and the values of upstream pressures were less than P'_e defined by Equation 70. Therefore, to demonstrate the effect of this, two different runs were conducted on sample LS-2 at a net rock confinement pressure of 400 psig by employing Equations 56 and 57. The results of the estimated parameters for these runs are summarized in Table 1 below.

TABLE 1

SUMMARY OF RESULTS FOR RUNS 1 AND 2 ON SAMPLE LS-2

Equation used	P_{nc} (psig)-Run No., Gas	Estimated Parameters		
		K(mds)	b(psia)	$F_B \times 10^9$
57	400-1, N_2	0.7767	6.349	16.40
56	400-2, N_2	0.9081	8.391	2.55
Relative change, %		14.47	24.33	-543.14

Alternatively, this effect can be demonstrated graphically by a Klinkenberg plot as presented in Figure 7. The rest of the runs in this study were conducted by employing the type curves generated from Equation 56 only.

B. Parameter Estimation

Five rock samples were studied in this work. The physical properties of these rock samples are summarized in Table 2. A total of 26 experimental runs were conducted on these rock samples and the raw data are presented in Tables C-1 through C-26 of Appendix C. The processed data, utilized in estimation of the parameters, and the results of estimation are presented in Tables D-1 through D-86 of Appendix D.

1. Choice of Parameter Estimation Technique

Applying the suggested data discrimination methods, the data points in viscous and in inertial regions, NV and NI, for three different runs were obtained and are presented in Table 3.

These results indicate that the data discrimination methods yield different results for all three runs. Therefore, one can not expect reliable estimates for the parameters K, b, and F_B from any graphical technique which has to employ one of these data discrimination techniques. Consequently, the accuracy of estimated parameters from the graphical or semigraphical techniques depend upon the data splitting method one prefers to use.

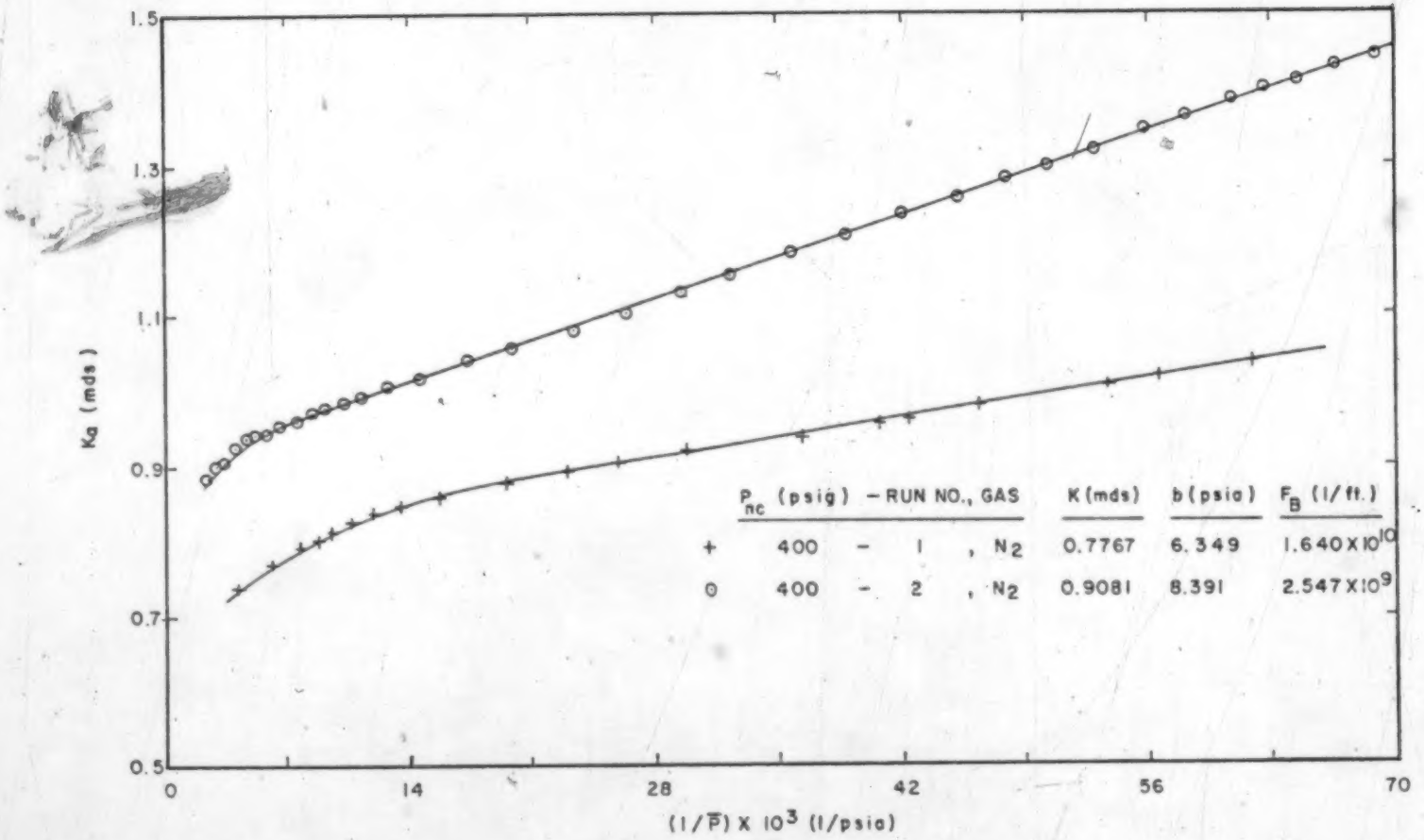


FIGURE 7 KLINKENBERG PLOTS FOR SAMPLE LS-2 RUNS 1 & 2

TABLE 2

PHYSICAL PROPERTIES OF THE ROCK SAMPLES

Sample No.	LS-1	LS-2	LS-3-A	SS-1-A	SS-2-A
Source	Imperial Oil Labs	Sharp Bros. Cut Stone Co.	Imperial Oil Labs	Vanscoy, Saskatchewan	
Description	Indiana Limestone	Indiana Limestone	Indiana Limestone	Blairmore Sandstones	
Bore Hole Radius (ft)	0.0150	0.02079	0.02120	0.01040	0.05440
Outer Radius (ft)	0.49990	0.48796	0.48983	0.49947	0.46290
Thickness (ft)	0.0750	0.07876	0.08018	0.08438	0.08460
Porosity (fraction)	0.1321	0.1095	0.1115	0.1266	0.1325
Average Uniaxial Compression Stress (psi)		unknown		10,000	3,150
Average Modulus of Elasticity (psi)		unknown		8.125×10^5	3.272×10^5

TABLE 3

RESULTS OF DATA DISCRIMINATION METHODS

Data Discrimination Method	Sample No., P_{nc}(psig)-Run No.					
	LS-1, 1000-3		LS-3-A, 400-1		SS-2-A, 800-4	
	NV	NI	NV	NI	NV	NI
Method 1	15	15	16	11	19	8
Method 2	10	20	7	20	9	18
Method 3	11	19	14	13	16	11
Table Number	D-17		D-47		D-86	

Theoretically, estimation of the visco-inertial flow parameters K, b, and F_B requires a method which is capable of performing a simultaneous three-dimensional search using Equation 48 with all of the data points. However, the graphical techniques are restricted to two-dimensional analysis where one of the parameters is either known, calculated independently, assumed or evaluated by data splitting and simultaneous plotting. Since the data discrimination methods appear to be inadequate, the usage of graphical methods should be avoided whenever possible.

Consequently, it was decided to estimate the unknown parameters, for the particular data set, in the following fashion:

1. First, construct the Klinkenberg plot so that wild data points can be checked.
2. Second, obtain the non-linear least squares estimates for parameters K, b and F_B by applying the Gauss-Newton Method to Equation 48.

2. Sensitivity Analysis

The unknown visco-inertial flow parameters, K, b, and F_B, for the particular rock sample, were estimated by using all the observed data points in applying the Gauss-Newton method to Equation 48. Sensitivity of these parameters to the number of experimental data points, used in their estimation, can be examined by comparing these values to those estimated with fewer data points. Using the data for Sample SS-2-A and run 2 of Table C-24, the values of K, b, and F_B were estimated and presented in Table 4. Examination of Table 4 yields the facts that the estimation of

1. All three parameters K, b, and F_B are not sensitive to the data points in the viscous region.
2. Both slippage coefficient b, and inertial coefficient F_B, are generally sensitive to the data points in the inertial region, i.e. relatively high flow rate data points, whereas the rock permeability, K, is less sensitive.

Furthermore, to see the relative changes in these sensitive parameters, with respect to base values, Table 5 was prepared.

TABLE 4

ESTIMATED VALUES QF K, b AND F_B FOR SAMPLE SS-2-A RUN 2

Estimated Parameters	Number of Data Points Used in Estimation: points from high flow rate side up, i.e. neglecting the points in the viscous region				All Points	points from low flow rate side up, i.e. neglecting the points in the inertial region			
	18	23	28	30	32 (base)	30*	28*	23*	18*
K(mds)	2.03	2.03	2.03	2.03	2.03	2.06	2.05	1.98	2.03
b(psia)	7.00	6.99	7.02	7.03	7.03	6.43	6.68	7.83	7.13
$F_B \times 10^{-9}$ (1/ft)	7.86	7.86	7.85	7.85	7.85	8.83	8.15	0.51	19.72

TABLE 5

SENSITIVITY OF K, b AND F_B TO THE NUMBER OF DATA POINTS

Number of Data Points	Relative Change in Parameters % K	b	F_B
30*	1.5	- 8.5	12.5
28*	1.0	- 5.0	3.8
23*	-2.5	11.4	- 93.5
18*	0.0	1.4	151.2

Consequently, it indicates that the accuracy of the estimated parameters depends on both the quality of experimental and the number of data points in the viscous and/or inertial regions.

C. Inertial Coefficient as a Property of the Rock and the Fluid

In order to determine whether the inertial coefficient, F_B, is dependent upon fluid properties as well as rock properties, flow tests were conducted on the rock samples LS-1 and SS-2-A using nitrogen, argon and methane as the flowing fluids. These gases were chosen on the basis of their availability and the significant differences in their viscosities and specific gravities.

The Klinkenberg plots for these rock samples and the related runs are shown in Figures 8 and 9. The results of estimated parameters for these runs are summarized and presented in Table 6. Examination of Table 6 indicates that unlike K, being a property of the rock alone,

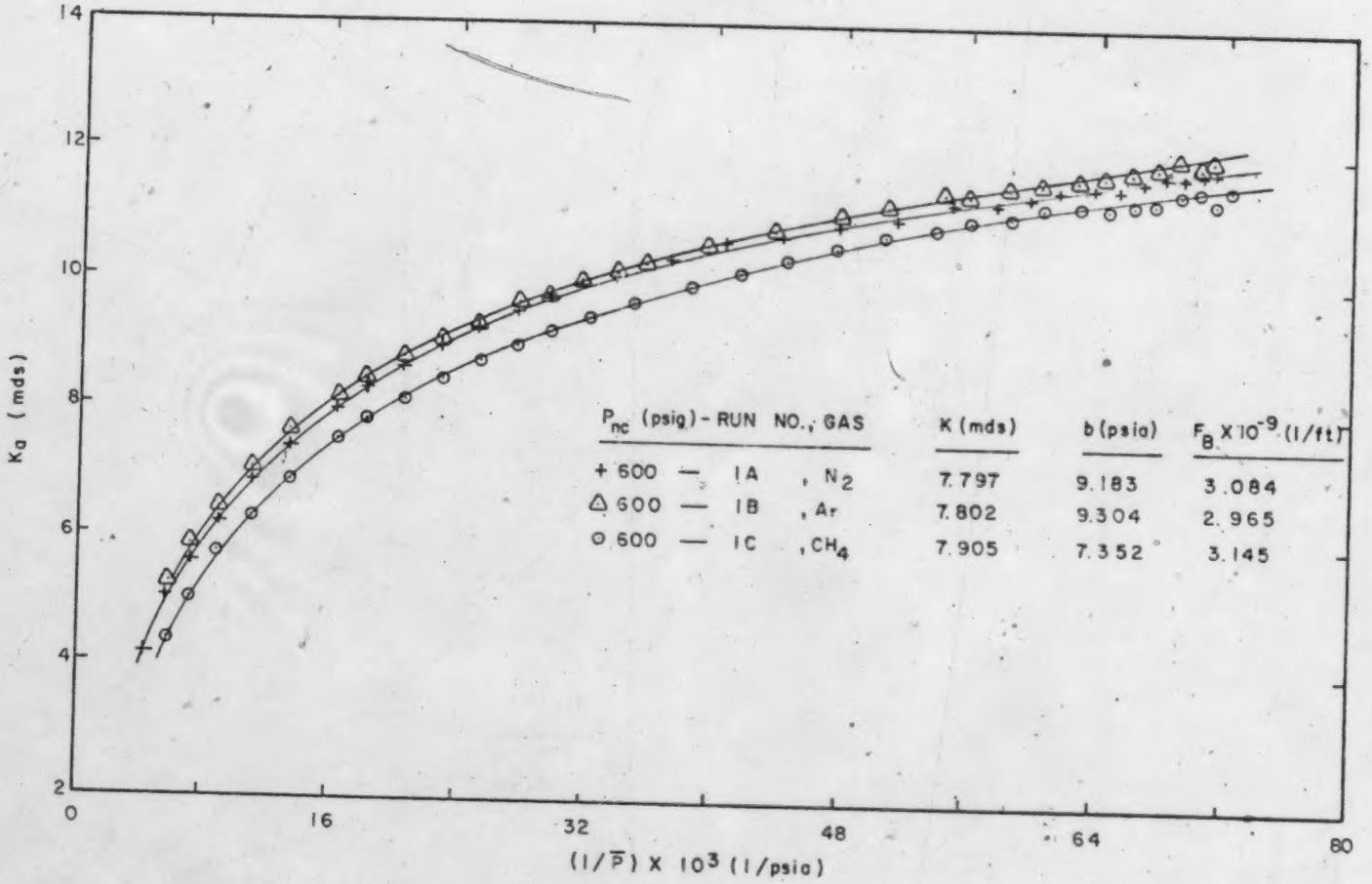


FIGURE 8 KLINKENBERG PLOTS FOR SAMPLE LS-1 RUNS 1A, 1B & 1C

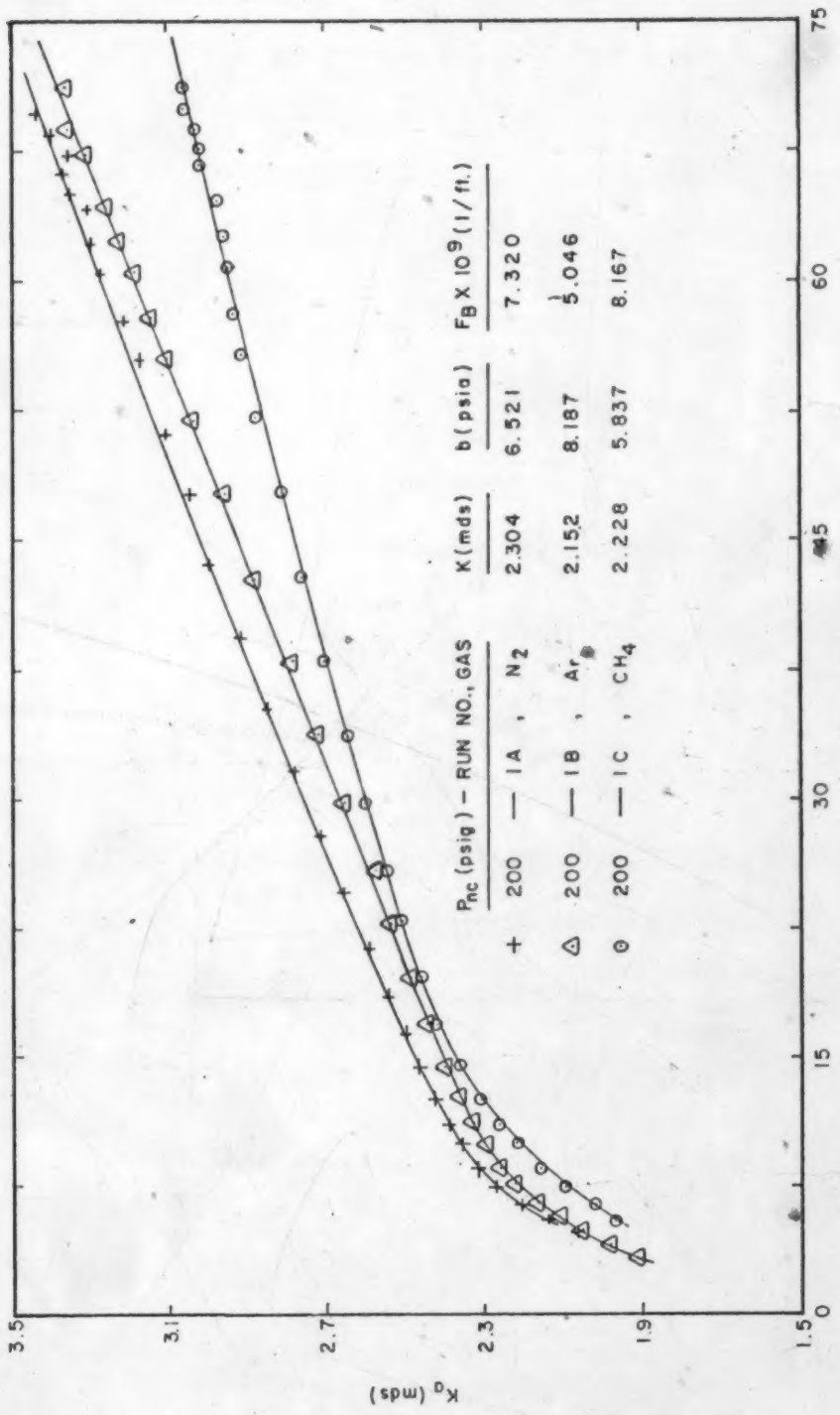


FIGURE 9 KLINKENBERG PLOTS FOR SAMPLE SS-2-A RUNS 1A, 1B & 1C

TABLE 6

SUMMARY OF RESULTS FOR RUNS WITH DIFFERENT GASES

Rock Sample, P_{nc}(psig)	Run No., Gas	Estimated Parameters K(mds)	b(psig)	$F_B \times 10^{-9}$ (ft)	Remarks*
LS-1, 600	1A , N_2	7.797	9.183	3.084	D-2
	1B , Ar	7.802	9.304	2.965	D-5
	1C , CH_4	7.905	7.352	3.145	D-8
SS-2-A, 200	1A , N_2	2.304	6.521	7.320	D-68
	1B , Ar	2.152	8.187	5.046	D-71
	1C , CH_4	2.228	5.837	8.167	D-74

* Table numbers of the non-linear parameter estimation results

the estimated values for b and F_B are significantly different for each gas and the rock sample. As a result of these differences in b and F_B for each gas and the rock sample, Table 7 was prepared to see the relationship between these parameters and the fluid properties, i.e. gas viscosity and specific gas gravity.

An examination of Table 7 clearly yields the following conclusions:

1. The slippage coefficient, b, is a property of both the rock and the fluid. This relationship may be represented as

$$b = f\left(\frac{1}{K}\,;\,\frac{\mu}{\sqrt{G}}\right) \tag{72}$$

This relationship is already known and supported by Equations 2 and 3 where the mean free path of the flowing gas, at $\bar{P}$ and $\bar{T}$, may be given by (94)

$$\bar{\lambda} = 0.375\left(\frac{\bar{\mu}}{\sqrt{G}}\right)\frac{\sqrt{R\bar{T}}}{\bar{P}} \tag{73}$$

However, examination of the data indicates that for both rock samples, b increases as the free mean path increases but in neither case is the relationship linear as suggested by Equations 3 and 73.

2. The inertial coefficient, F_B, appears to be a property of the rock as well as the fluid. This relationship may be expressed as

$$F_B = f\left(\frac{1}{K}\,,\,\frac{1}{\phi}\;;\;\frac{1}{\mu}\,,\,\frac{1}{G}\right) \tag{74}$$

TABLE 7

FLUID PROPERTIES AND ESTIMATED VALUES OF b AND F_B

Gas, Run No.	Gas Viscosity and Gravity @ 14.7 psia and 60°F			b(psia)		$F_B \times 10^{-9}$ (1/ft)	
	$\mu \times 10^5$ (cps)	$G(G_{air} = 1)$	$\frac{\mu}{\sqrt{G}} \times 10^5$	LS-1	SS-2-A	LS-1	SS-2-A
CH_4, 1C	1106	0.5537	1486.34	7.35	5.84	3.15	8.17
N_2, 1A	1776	0.9672	1809.86	9.18	6.52	3.08	7.32
Ar, 1B	2262	1.3788	1926.38	9.30	8.19	2.97	5.05

The Figures 10 and 11 demonstrate this dependence of F_B on the gas viscosity and specific gravity.

Consequently, within the range studied, it is appropriate to say that like b, F_B appears to be a property of the fluid as well as the rock. However, since the permeability varied from fluid to fluid and b did not vary linearly, the possibility that the observed variation of F_B from fluid to fluid was due to experimental error can not be ruled out. Hence, further investigations, covering observations in a wider range, are required to define this dependence more clearly.

D. Effect of Confinement Pressure on Parameters

In order to determine the effect of confinement pressure on the parameters K, b, and F_B, flow tests under different net confinement pressures were conducted and the results are examined in the following subsections.

1. Problems Encountered

The Klinkenberg plots for these runs, with the processed data and the related parameter estimation results presented in Tables D-1 through D-86, are shown in Figures 12 through 19. These figures show that there are different behavior of the Klinkenberg plots for different runs which may be explained in the following manner:

(a) Runs with Flow Test Method Case I

These runs yield three different types of behavior in the Klinkenberg plots,

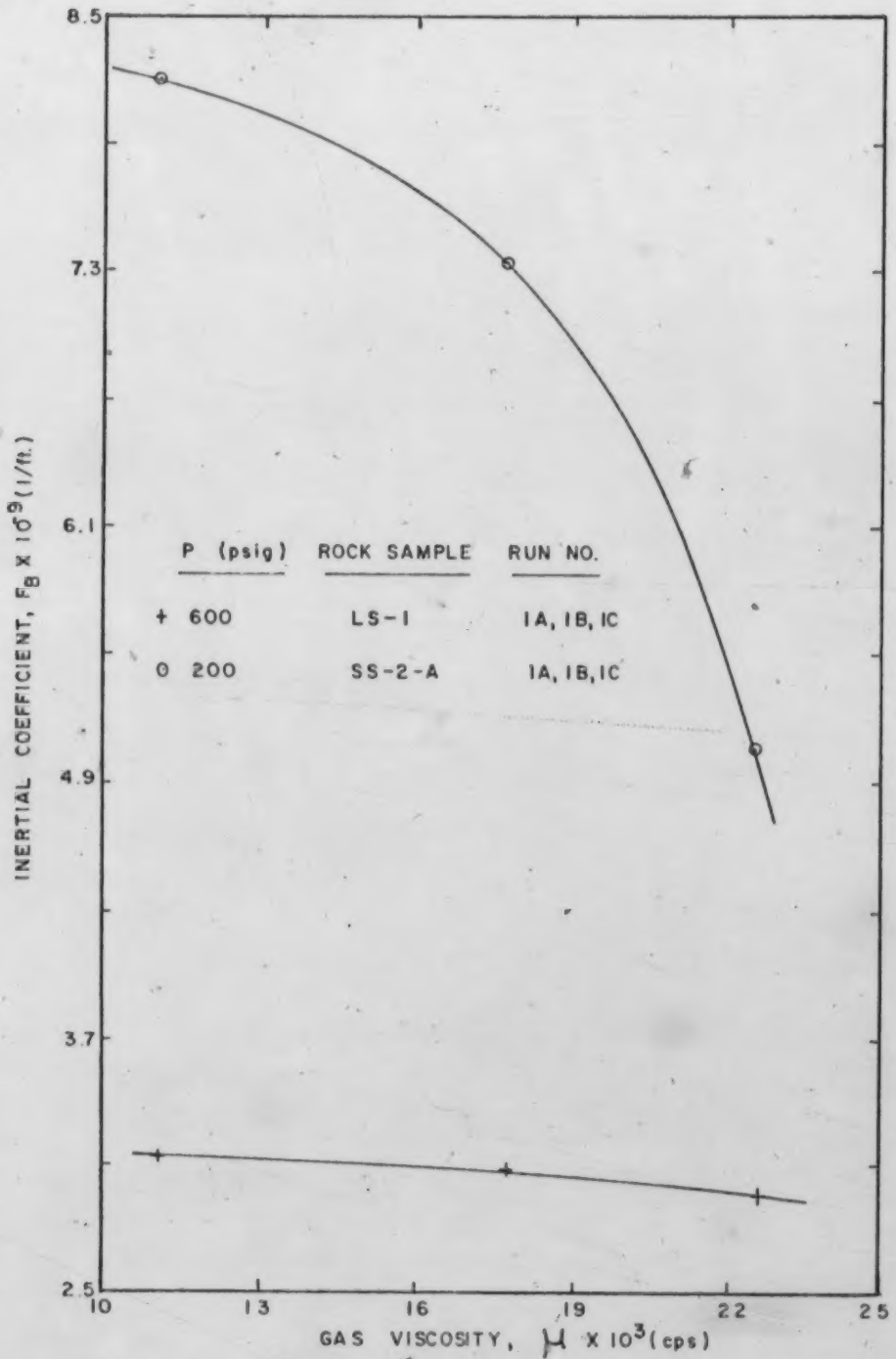


FIGURE 10 F_B AS A FUNCTION OF μ

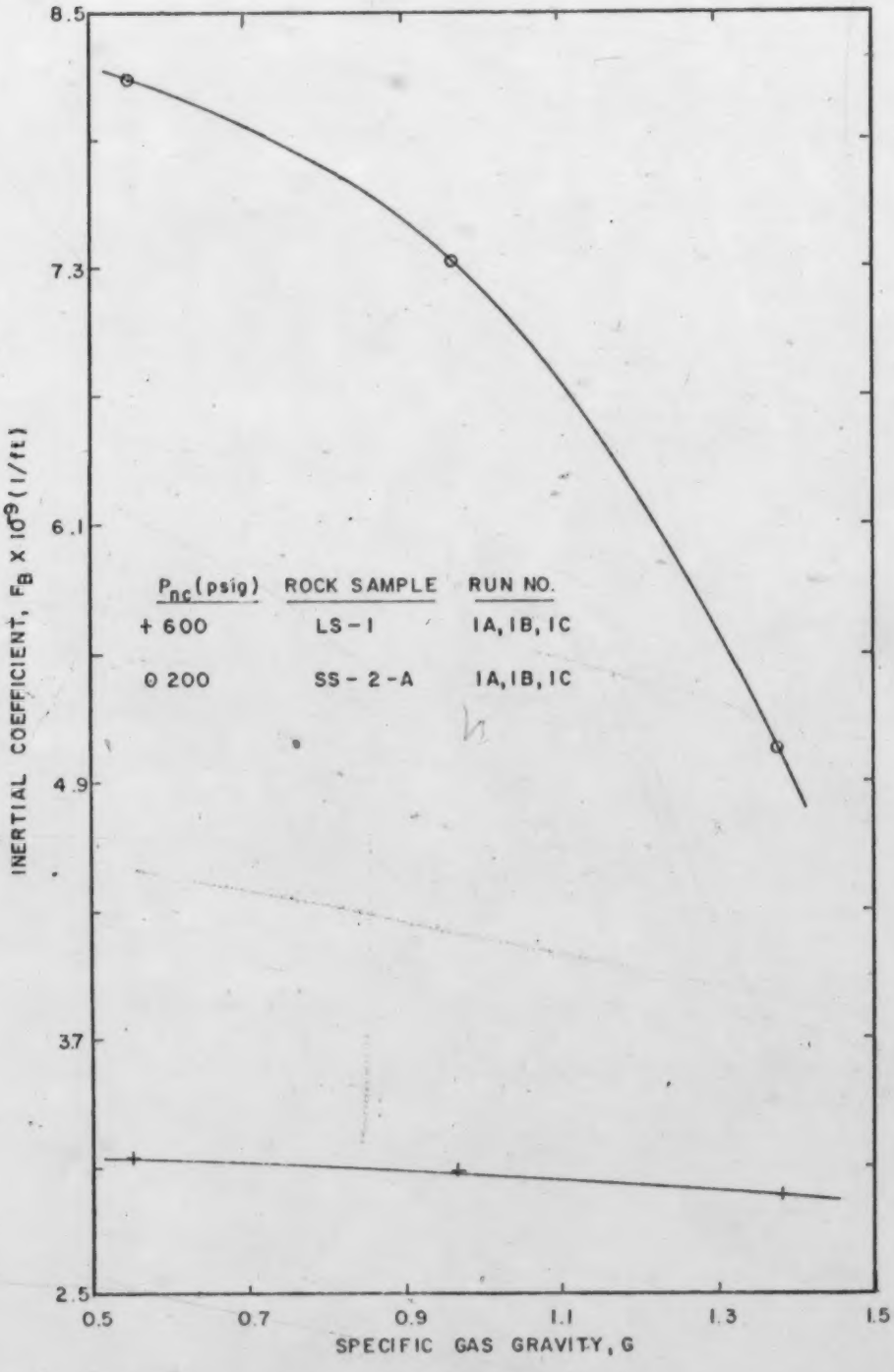


FIGURE 11 F_B AS A FUNCTION OF G

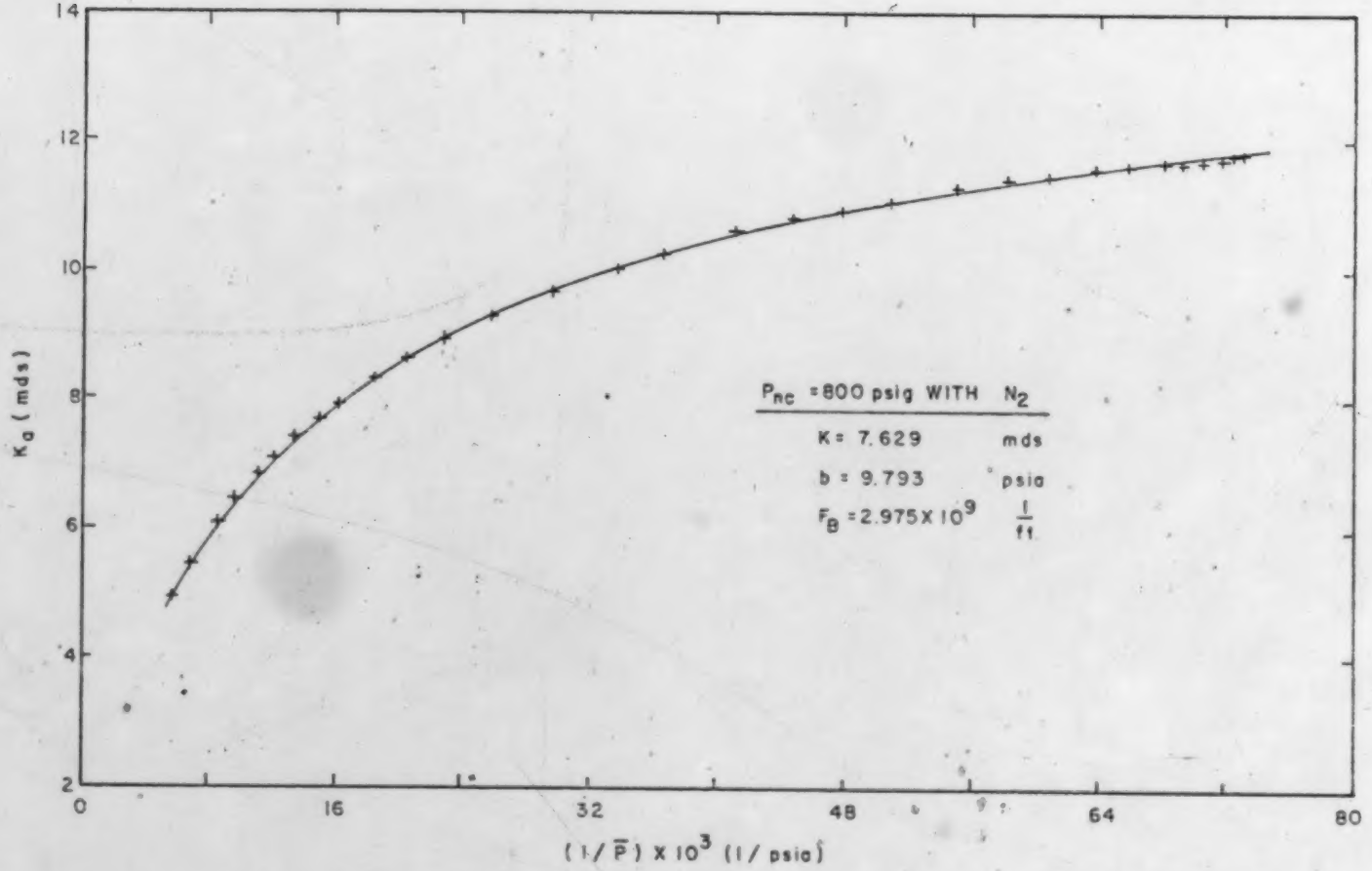


FIGURE 12 KLINKENBERG PLOT FOR SAMPLE LS-1 RUN 2

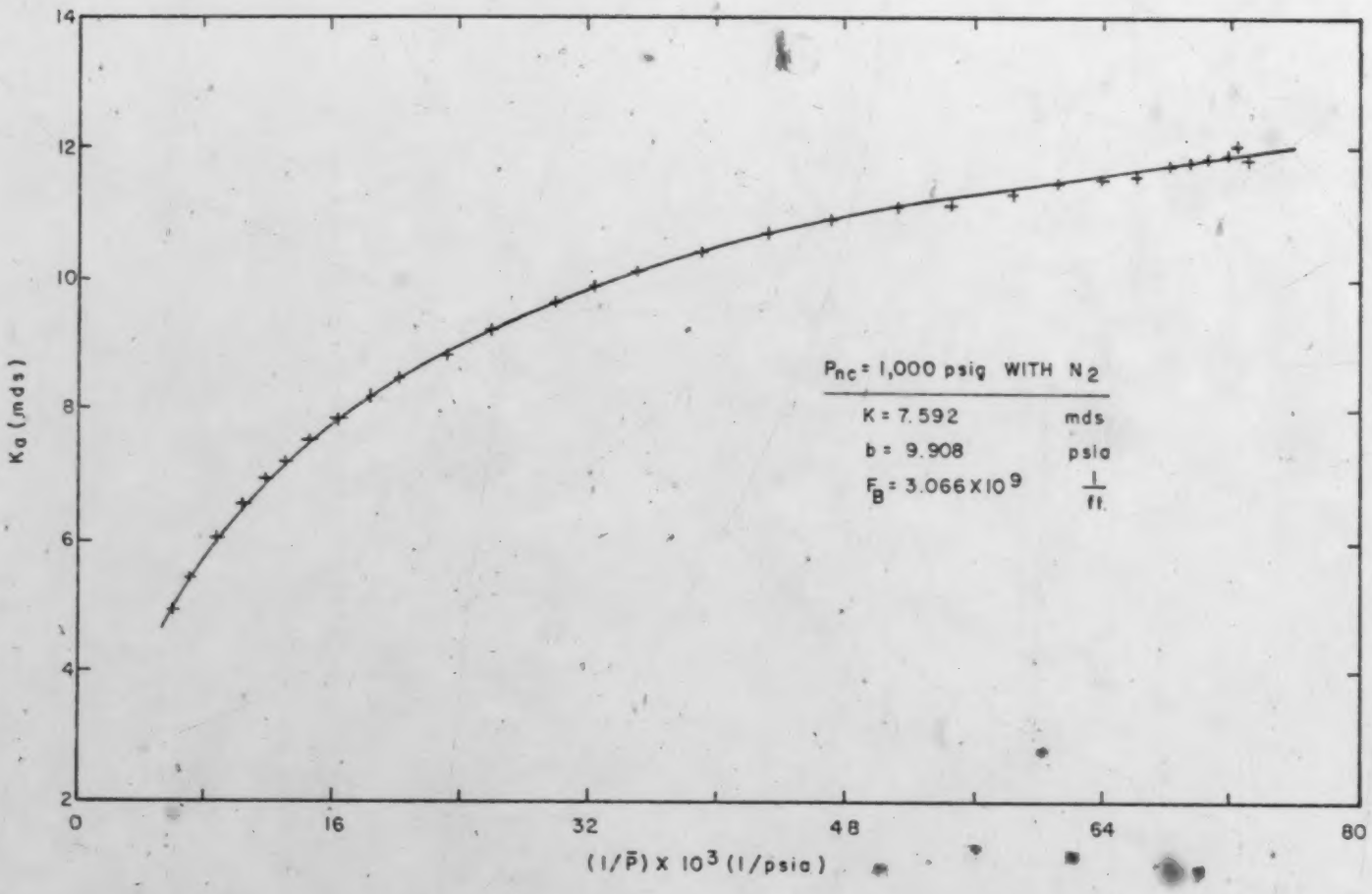


FIGURE 13 KLINKENBERG PLOT FOR SAMPLE LS-1 RUN 3

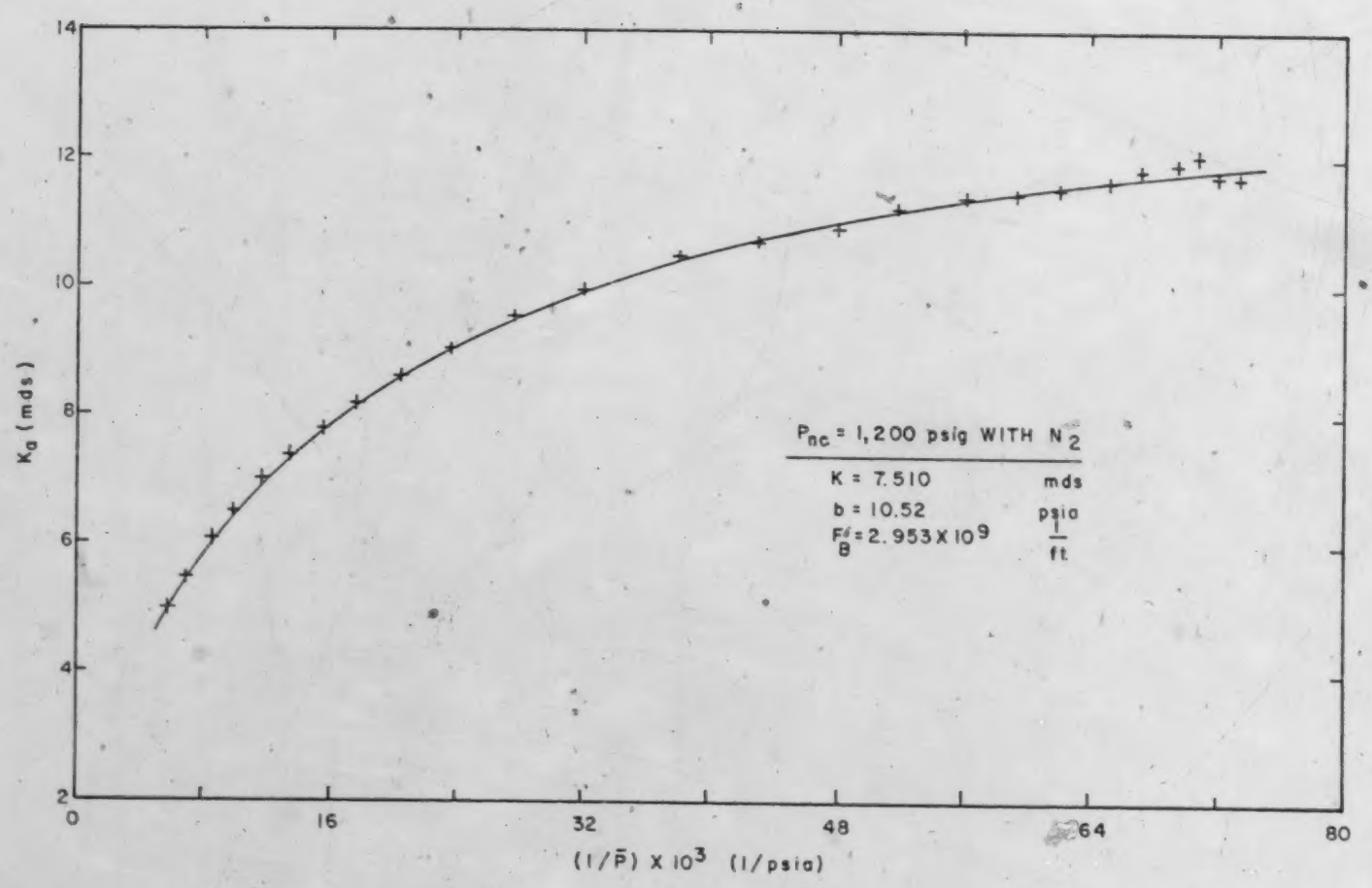


FIGURE 14 KLINKENBERG PLOT FOR SAMPLE LS-1 RUN 4

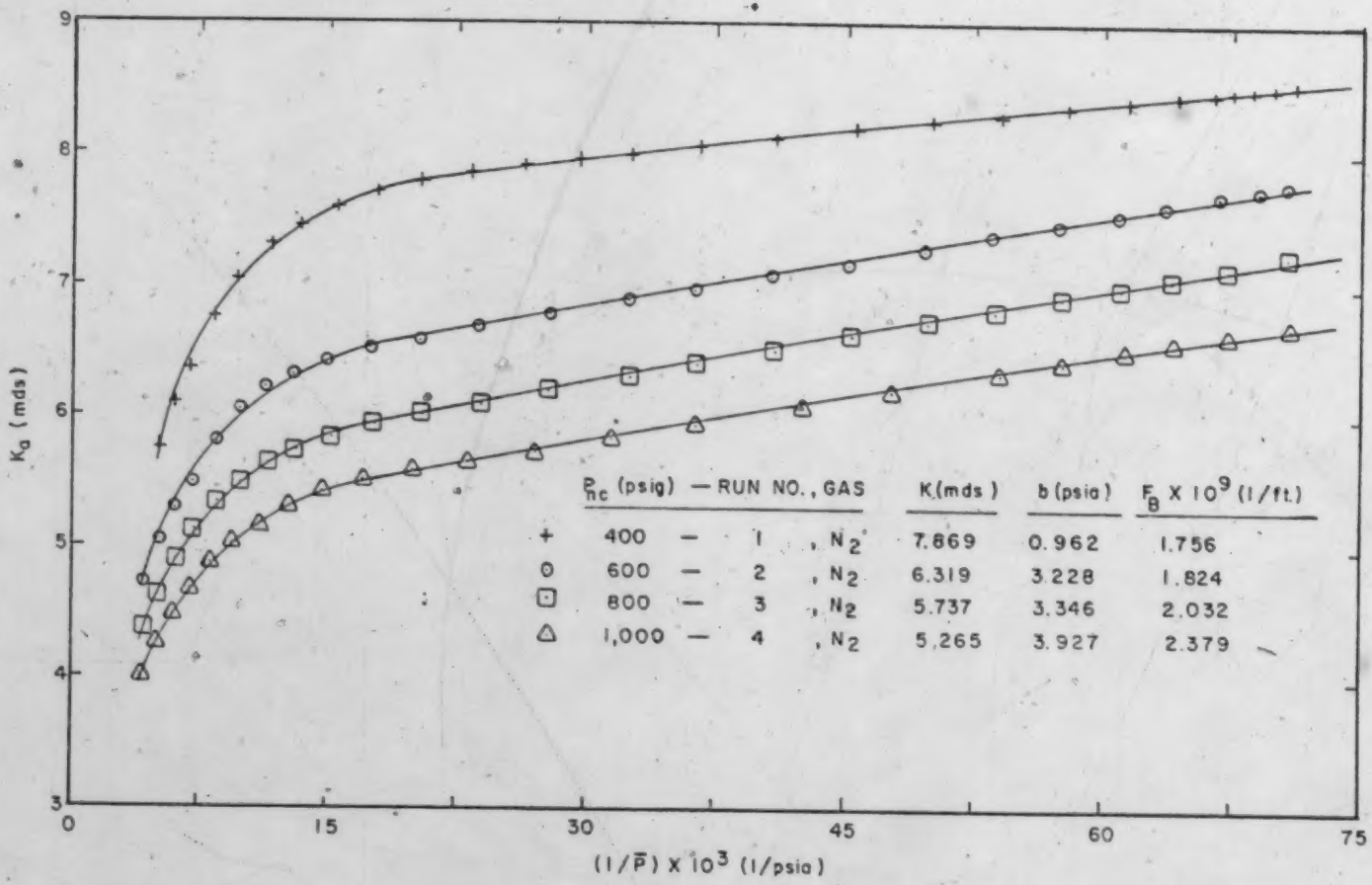


FIGURE 15 KLINKENBERG PLOTS FOR SAMPLE LS-3-A RUNS 1,2,3 & 4

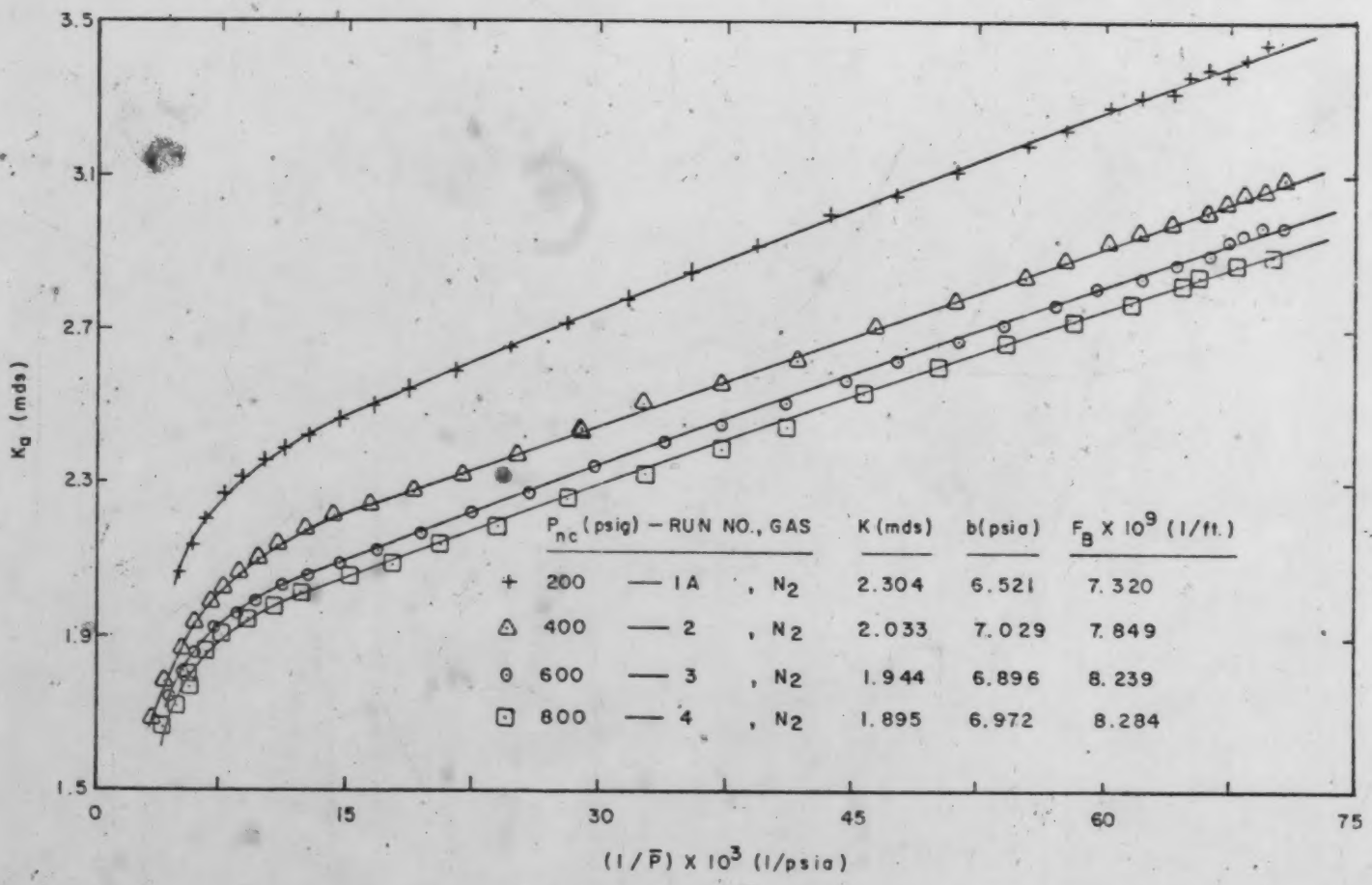


FIGURE 16 KLINKENBERG PLOTS FOR SAMPLE SS-2-A RUNS 1A, 2, 3 & 4

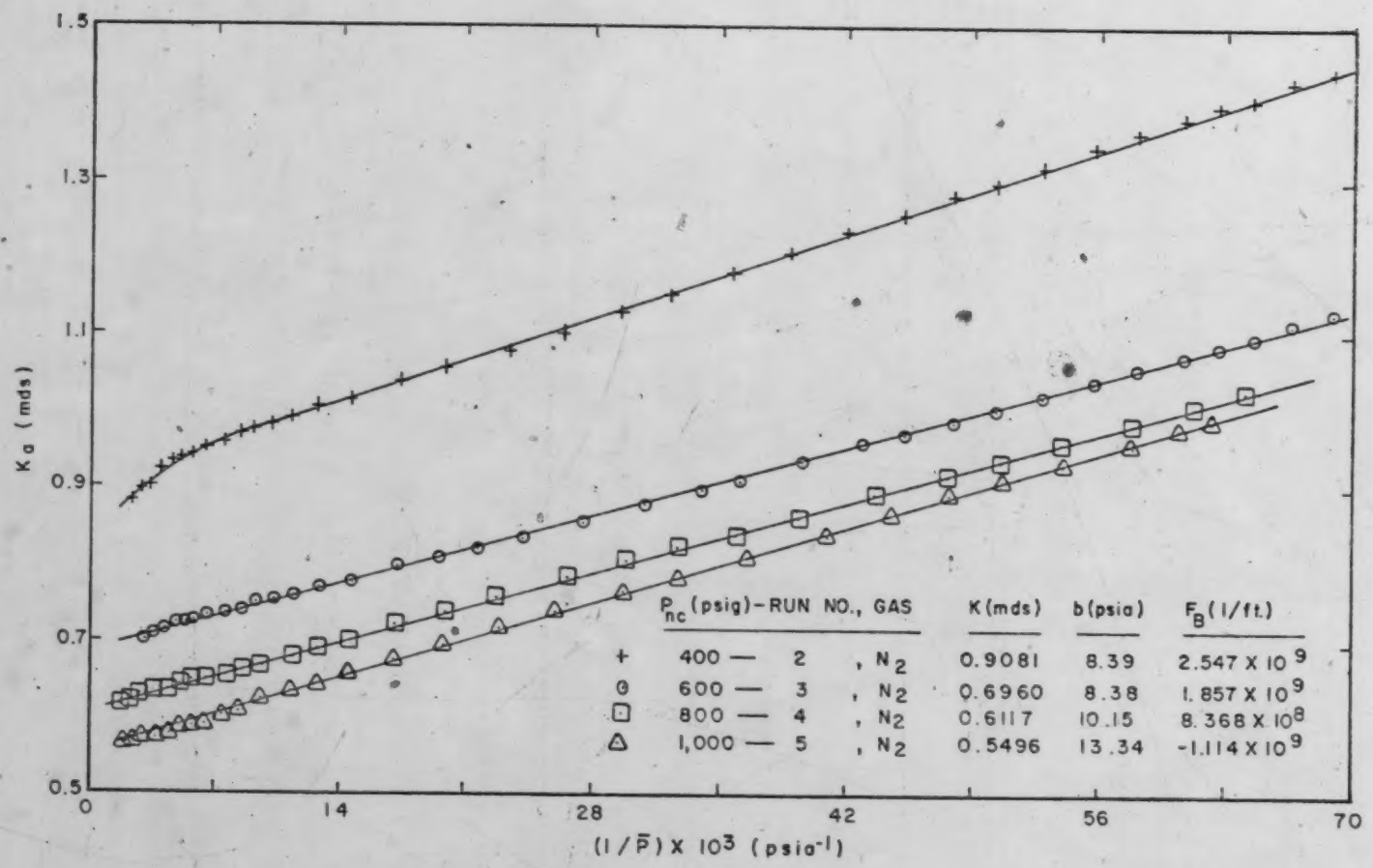


FIGURE 17 KLINKENBERG PLOTS FOR SAMPLE LS-2 RUNS 2,3,4&5

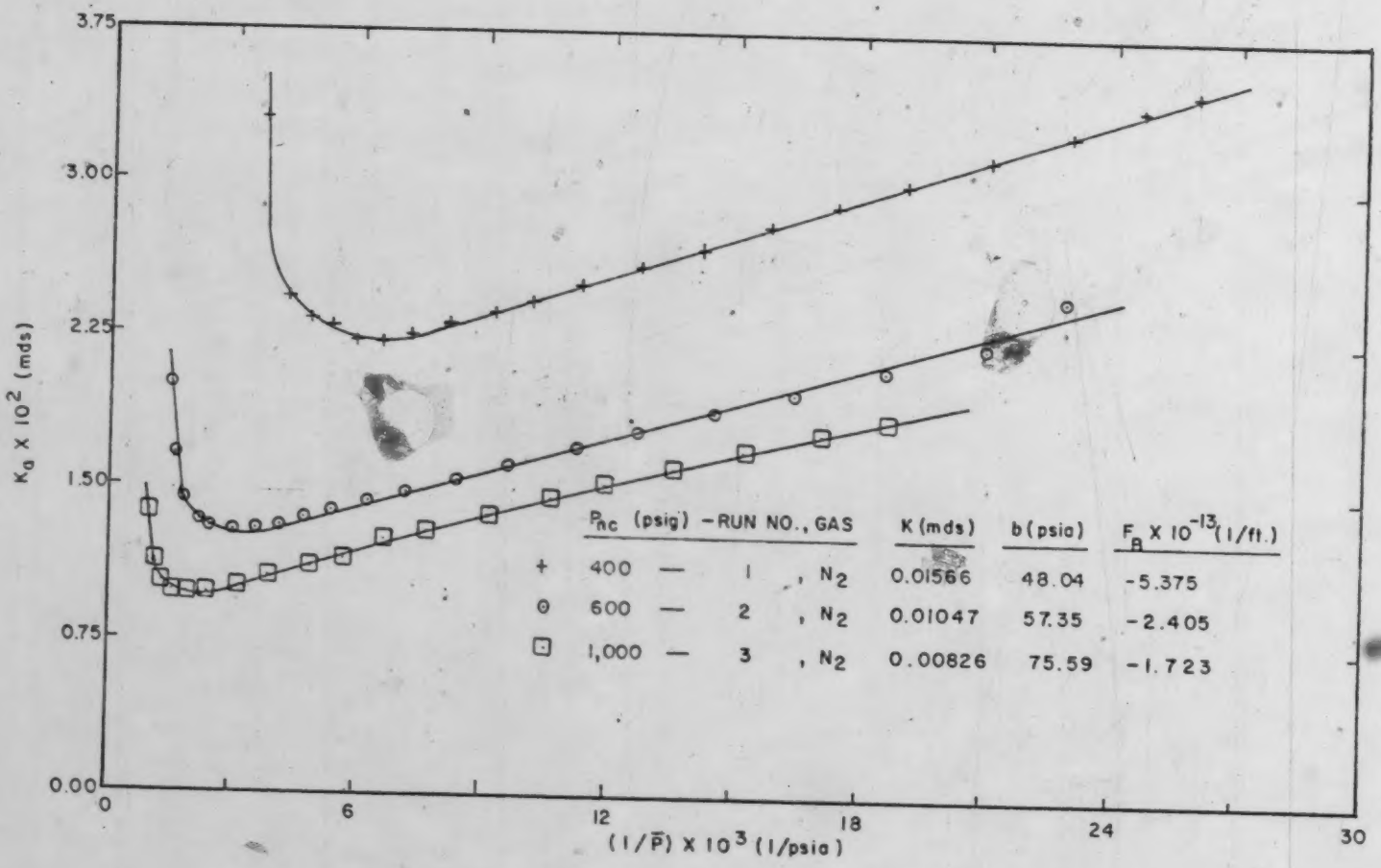


FIGURE 18 KLINKENBERG PLOTS FOR SAMPLE SS-1-A RUNS 1,2&3

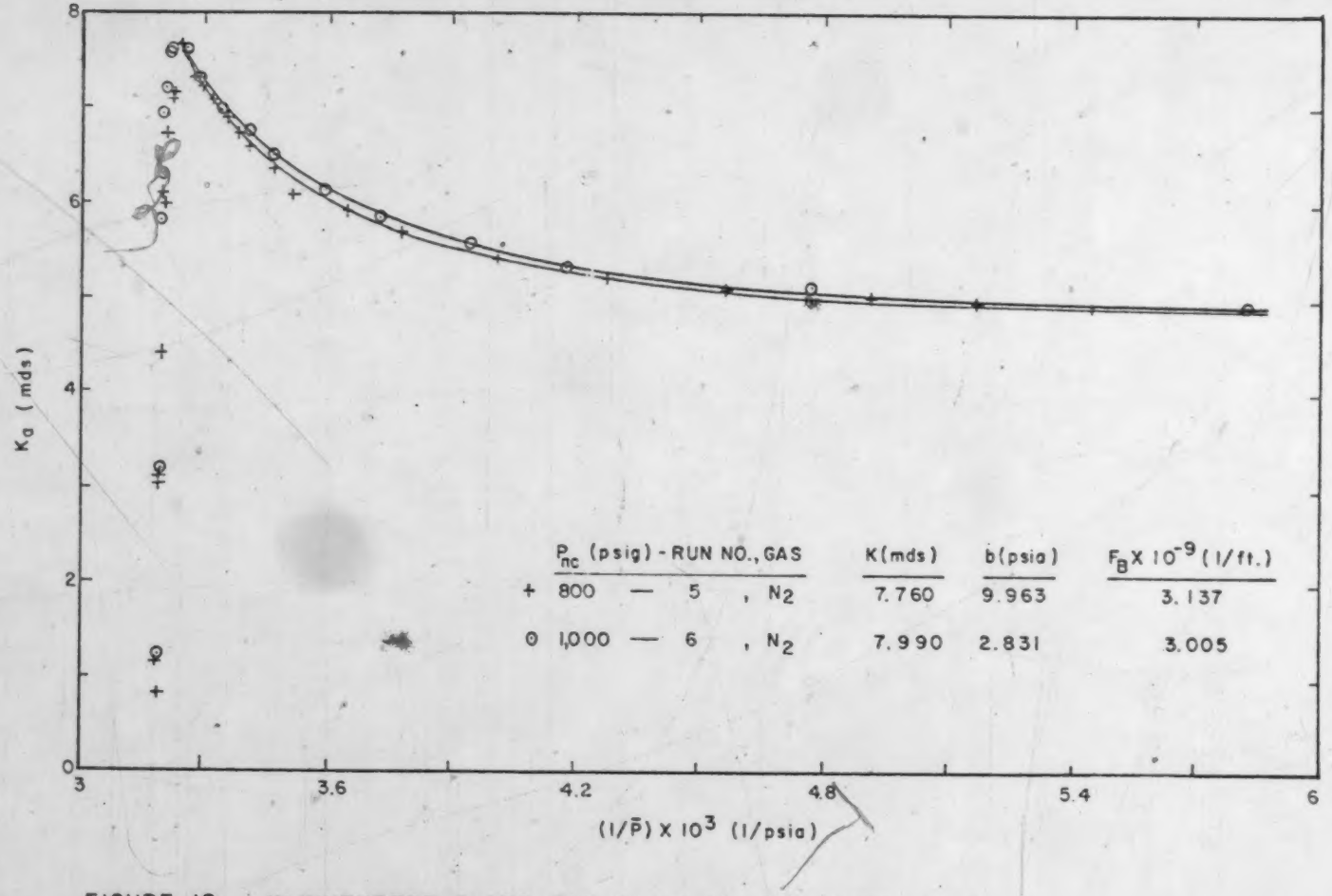
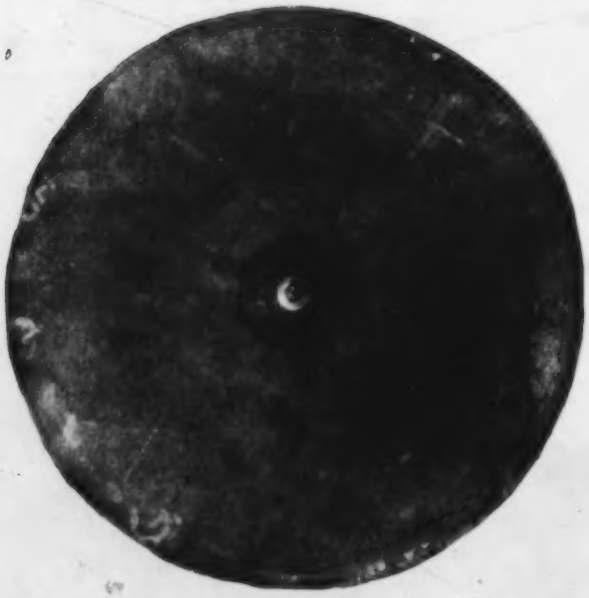


FIGURE 19 KLINKENBERG PLOTS FOR SAMPLE LS-1 RUNS 5 & 6

1. Runs with the expected behavior of the Klinkenberg plots with a straight line and a concave-downward portion as illustrated in Figures 12 through 16.
2. Runs with straight line Klinkenberg plots with almost no concave-downward portions as shown in Figure 17. Examination of Figure 17, the Klinkenberg plots without inertial regions, yielded a problem which led to the discovery of a rupture in the bottom rubber gasket at the borehole, as shown in Plate 1(A). This rupture suggests that blow-by occurred between the piston and the rubber gasket. However, the possibility exists that blow-by also occurred at the other three gasket surfaces.
3. Runs with anomalous behavior of the Klinkenberg plots, with systematic concave-upward instead of concave-downward portions, as shown in Figure 18. At the end of run 1 on rock sample SS-1-A, a sudden increase in the measured flow rate was noted in the last data point. It was thought that this increase was due to fracturing of the rock sample similar to one particular failure as shown in Plate 1(B). The flow cell was dismantled and an examination indicated that the rock sample was not damaged but that there was a small perforation in the bottom rubber gasket at the borehole.



(A) RUPTURE IN BOTTOM RUBBER GASKET AT THE BOREHOLE.

(B) TRACES OF FRACTURED ROCK SAMPLE ON THE BOTTOM RUBBER GASKET

PLATE 1. PHOTOGRAPHS OF RUBBER GASKETS USED FOR ROCK SAMPLE CONFINEMENT.

In order to prevent another rupture in the new gasket, runs 2 and 3 on this sample were conducted after placing a small circular stainless steel sheet between the rubber and the rock sample at the borehole. As it is evident from the Klinkenberg plots for these runs, as shown in Figure 18, this arrangement did not change the systematic behavior of the plots even though there was no rupture in the rubber gasket.

As a result, there are many possible reasons for the unusual behavior of the Klinkenberg plots, namely

(i) blow-by

(ii) inadequate definition of the net rock confinement pressure

(iii) lack of radial confinement of the rock samples

(iv) permeability of the rubber gaskets, and

(v) experimental errors.

Furthermore, it may also be true that the runs conducted on samples LS-1, LS-3-A and SS-2-A are not exceptional but are the ones where the effects of those aforesaid possibilities are smaller as compared to the inertial effects. Since these rock samples have higher permeabilities, ranging from 2 to 8 mds, than SS-1-A and LS-2, i.e. 0.01 mds and 1 mds respectively, it appears that blow-by and/or other possibilities did not play any major role during runs conducted on samples LS-1, LS-3-A and SS-2-A.

(b) Runs with Flow Test Method Case II

Only runs 5 and 6 were conducted, using this method, on rock sample LS-1. The Klinkenberg plots for these runs are presented in Figure 19. This figure illustrates several peculiarities. First, each plot has a portion showing sudden and continuous reduction in permeability when the flowing mean gas pressures are close to predetermined upstream pressure, i.e. P_e = 300 psig. Secondly, each plot contains a section where permeability increases with increasing mean flowing pressure and finally the position of the two curves is opposite to that expected.

The reason for these peculiarities may be explained in the following manner:

1. Sudden Sharp Decrease in Permeability

   It is appropriate to restate that these runs 5 and 6 on rock sample LS-1 were conducted under constant net confinement pressures of 800 and 1000 psig, respectively, while maintaining a constant upstream pressure of 300 psia. Flow rate observations became troublesome, i.e. very unstable, for downstream pressure over 290 psig during these runs. Therefore, the error in flow rate measurements is thought to be the cause for this sudden sharp decrease in permeability.

   In order to determine the possible error in the flow rate measurements, one may try to predict

flow rates by using Equation 48 together with the flow data for these runs, Tables C-7 and C-8, provided K, b, and F_B are known. At this point it was assumed that the parameters estimated for sample LS-1 in runs 2 and 3 were true parameters. Since these runs were also conducted under the same net confinement pressure of 800 and 1000 psig and on the same sample, this assumption seems reasonable. Percent error between the measured and predicted flow rates for 5 different runs were estimated and presented in Table 8. Errors in this table tend to support the belief that these flow rate measurements were erroneous.

2. Increase in Permeability and Shape Reversal

One may think this behavior is due to either blow-by or the stress state of the rock sample, i.e. inadequate confinement pressure. However, examination of these sections of Figure 19 shows reversal in shape, i.e. higher permeability values at 1000 psig net confinement pressure than 800 psig, which is opposite to that expected. On the basis of these observations, it is felt that there is insufficient information available to explain this behavior at this time. Therefore, it is suggested that runs with flow test method Case II should be reexamined in greater detail so as to explain the behavior in such runs.

TABLE 8

ERRORS IN FLOW RATE MEASUREMENTS

Rock Sample	P_{nc}(psig)-Run No., Test Method	Error, % (min., max.)	Remarks*
LS-1	800-5, Case II	0.22 , -855	D-24
	1000-6, Case II	-0.15 , -535	D-28
	800-2, Case I	0.08 , - 11	D-13
LS-3-A	1000-4, Case I	0.008, - 1.15	D-57
SS-2-A	600-3, Case I	0.002, 1.5	D-82

* Table numbers of the results for flow rate predictions

As a result of this study, anomalous behavior of the Klinkenberg plots was found in runs

(i) on low permeability rock, i.e. the samples LS-2 and SS-1-A

(ii) having high mean flowing gas pressures, i.e. the runs 5 and 6 on sample LS-1.

2. Confinement Pressure Effect

Non-linear estimates for the parameters K, b and F_B under different net confinement pressures are presented in Table 9. Using the appropriate values in Table 9, the effect of confinement pressure on parameters was examined and is presented in the following fashion:

1. Effect of confinement pressure on the permeability is shown by plotting permeability factor f_K, versus net confinement pressure as presented in Figure 20. Examination of this figure yields that the absolute permeability, K, decreases as the net rock confinement pressure increases. The effect is more severe in low permeability rock samples than in high permeability ones. These results support the findings reported by Fatt (95). It should be noted that this interpretation is valid only for the same kind of rock samples, i.e. sandstones or limestones etc.

2. The confinement pressure effect on the slippage coefficient, b, is shown in Figure 21. Unlike K,

TABLE 9

VALUES OF K, b AND F_B FOR THE ROCK SAMPLES

Rock Sample	P_{nc}(psig) - Run No.	K(mds)	b(psia)	$F_B \times 10^{-9}$ (1/ft)	ϕ(%)	Remarks*
LS-1	600 - 1A	7.797	9.183	3.084	9.7	D-2
	800 - 2	7.629	9.793	2.975	9.5	D-11
	1000 - 3	7.592	9.908	3.066	9.2	D-15
	1200 - 4	7.510	10.52	2.953	9.1	D-19
LS-3-A	400 - 1	7.869	0.962	1.756	61.9	D-45
	600 - 2	6.319	3.228	1.824	26.8	D-49
	800 - 3	5.737	3.346	2.032	24.5	D-52
	1000 - 4	5.265	3.927	2.379	20.0	D-55
SS-2-A	200 - 1A	2.304	6.521	7.320	7.3	D-68
	400 - 2	2.033	7.029	7.849	6.6	D-77
	600 - 3	1.944	6.896	8.239	6.5	D-80
	800 - 4	1.895	6.972	8.284	6.4	D-84
LS-2	400 - 2	0.9081	8.391	2.547	11.6	D-33
	600 - 3	0.6960	8.376	1.857	14.0	D-36
	800 - 4	0.6117	10.15	0.8368	19.8	D-39
	1000 - 5	0.5496	13.34	-1.114	13.9	D-42
SS-1-A	400 - 1	0.0156	48.04	-53750	0.009	D-59
	600 - 2	0.0105	57.35	-24050	0.013	D-62
	1000 - 3	0.0083	75.59	- 1723	0.013	D-65

* Table numbers of the non-linear parameter estimation results.

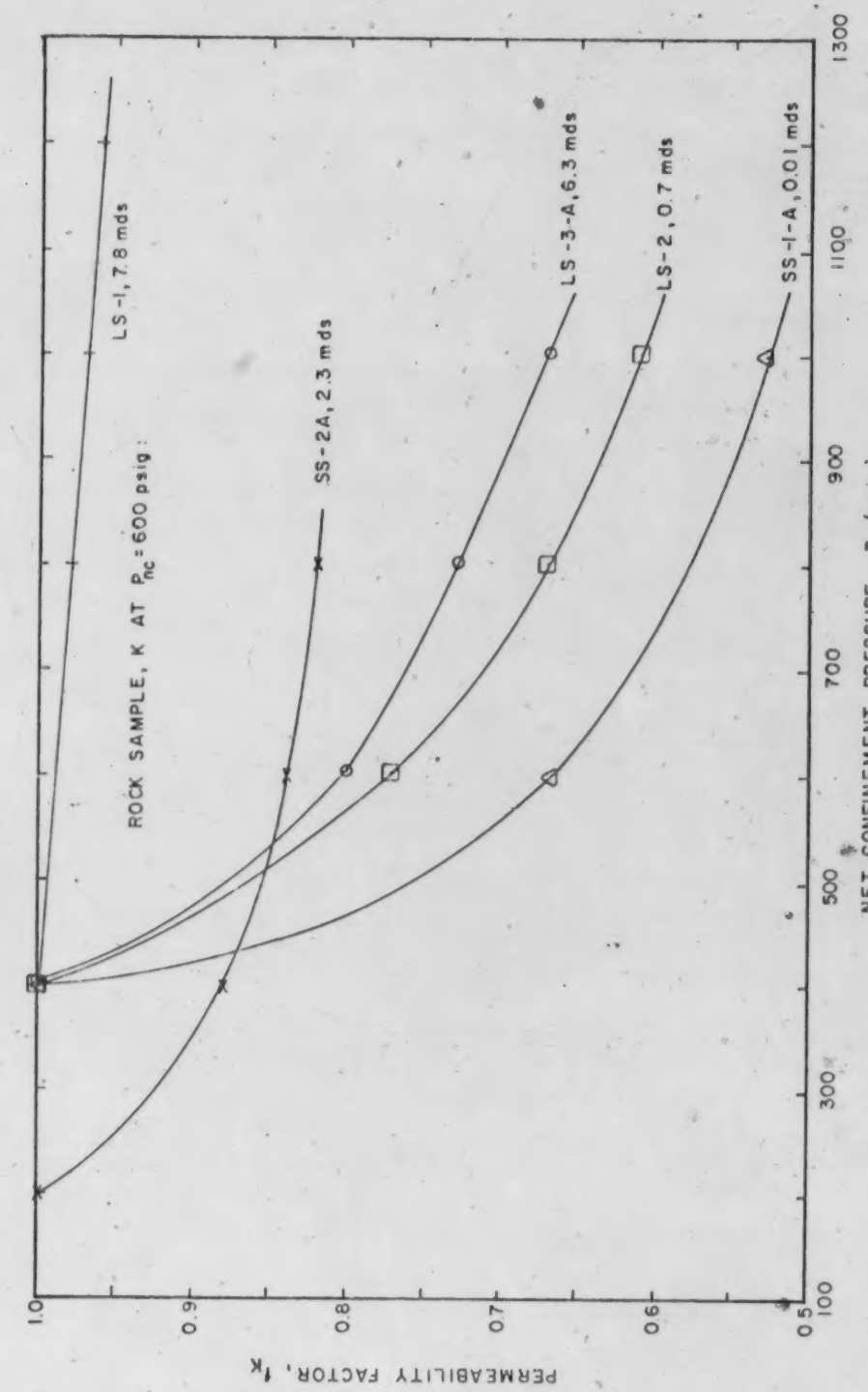


FIGURE 20 CONFINING PRESSURE EFFECT ON K

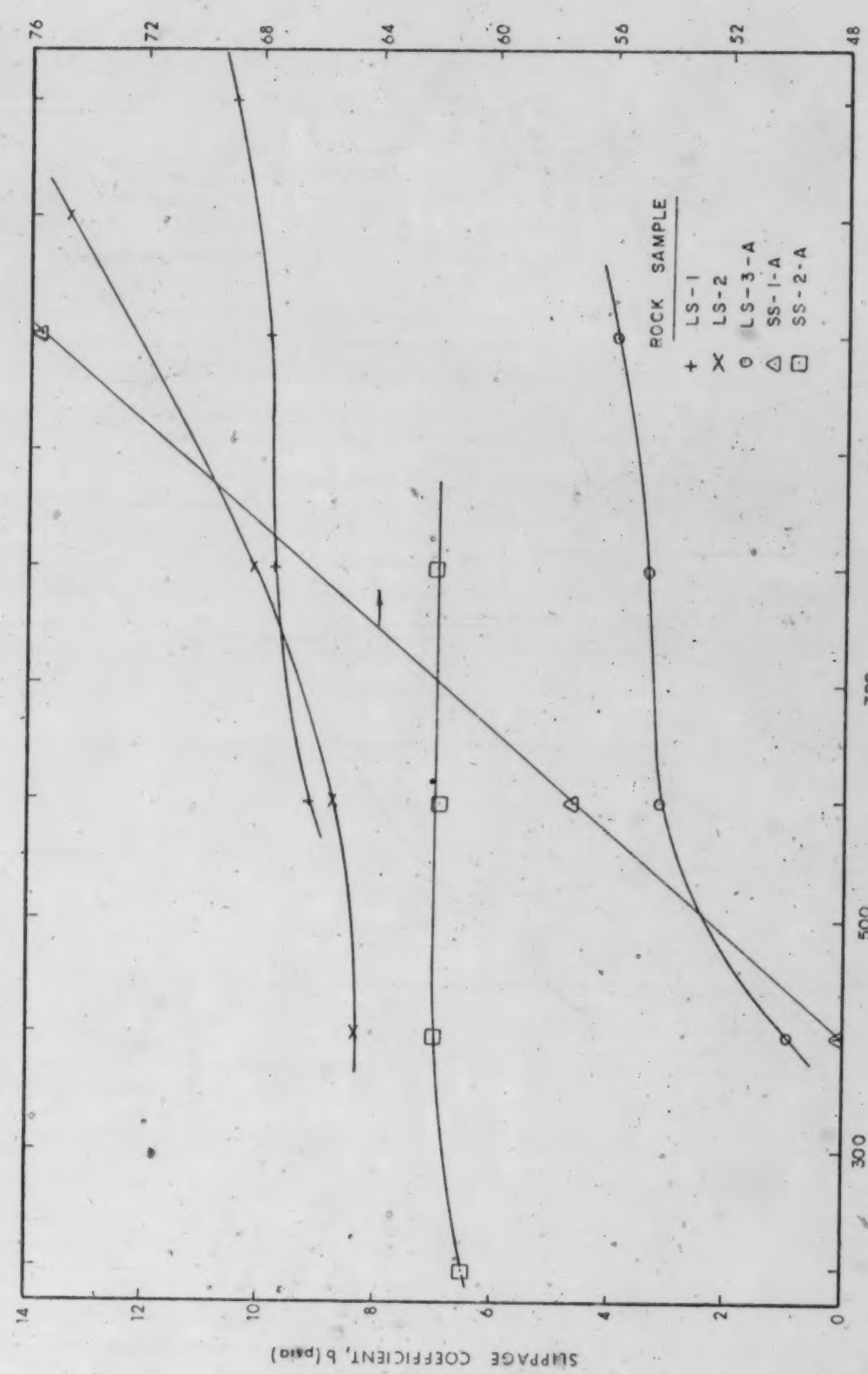


FIGURE 21 CONFINING PRESSURE EFFECT ON b

values of b increase as the net confinement pressure increases. Furthermore the confinement pressure effect on b is more pronounced in low permeability rock samples, i.e. LS-2 and SS-1-A.

3. As a result of Figure 22, one may conclude that the inertial coefficient, F_B, increases as the net confinement pressure increases. However, examination of Figure 23 reveals contradictory results. The behavior of the curves in Figure 23 is caused by the inaccurate estimated values of inertial coefficient using the erroneous flow data of the runs conducted on the rock samples LS-2 and SS-1-A. As previously concluded, the blow-by effects increasingly became important and overcome the inertial effects for the rock sample LS-1, i.e. upper curve in Figure 23. However, for sample SS-1-A the blow-by effects are much more, as compared to the inertial effects, and steadily decrease as the net confinement pressure increases to 1000 psig, i.e. the lower curve in Figure 23 levels off.

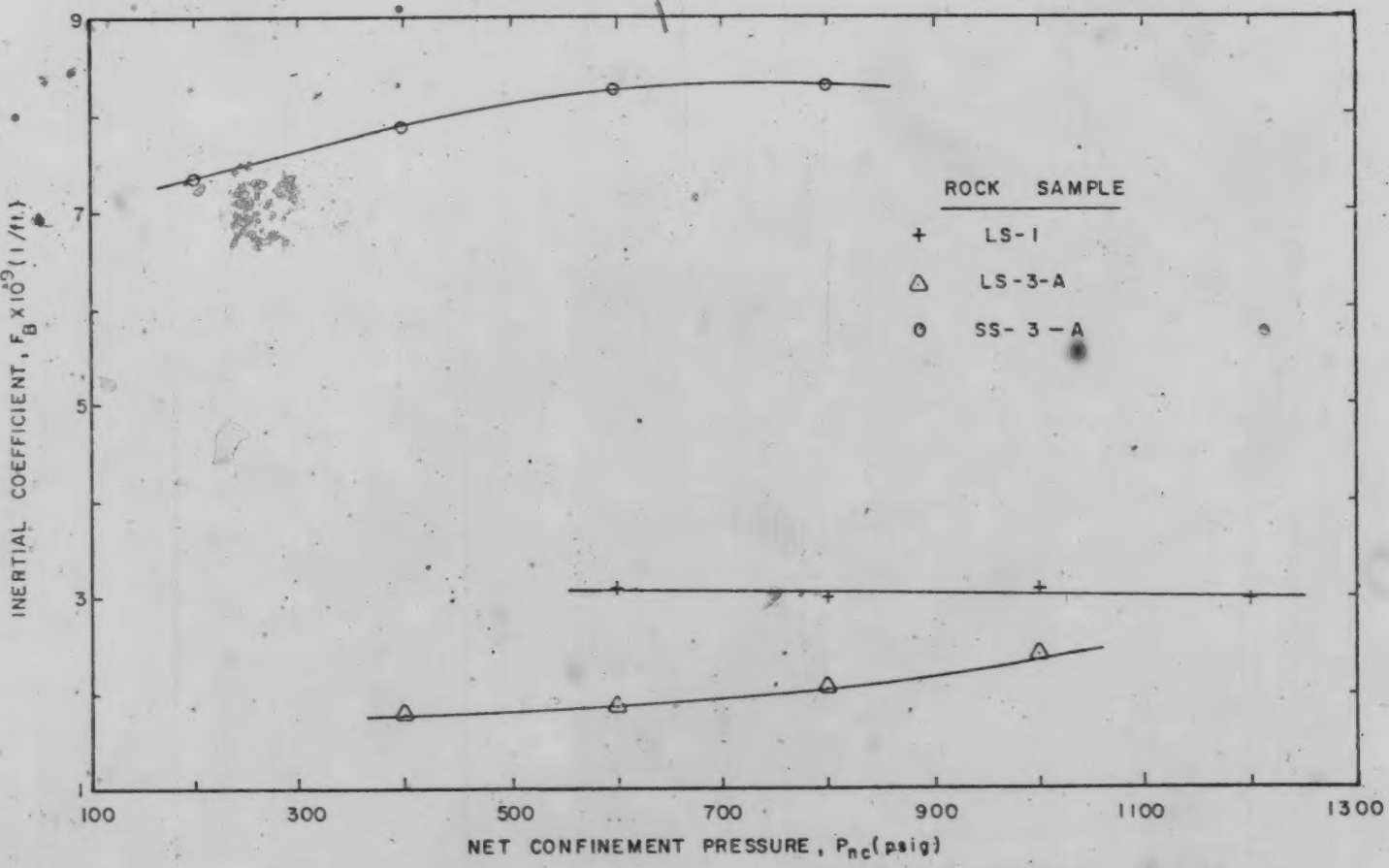


FIGURE 22 CONFINING PRESSURE EFFECT ON F_B

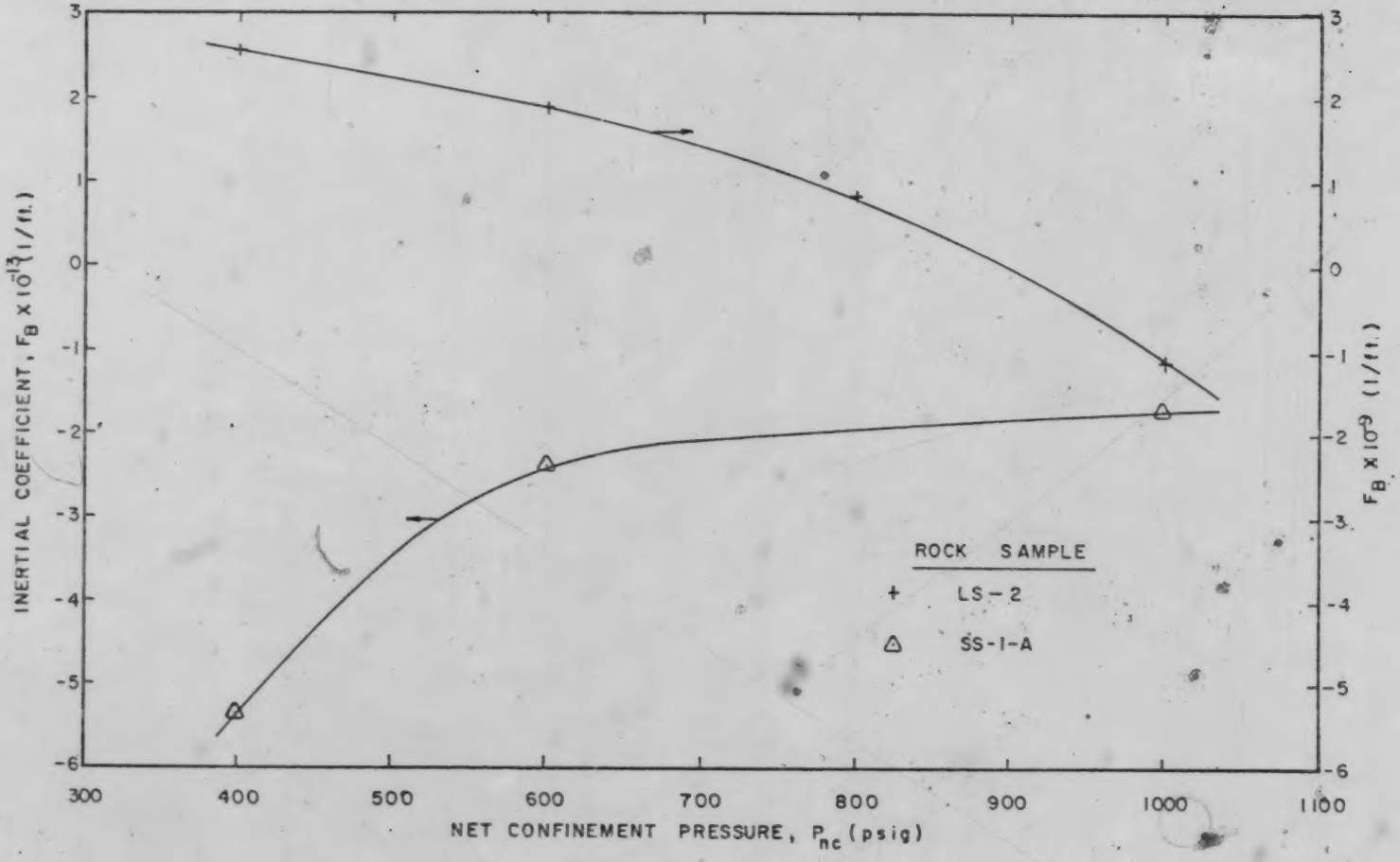


FIGURE 23 CONFINING PRESSURE EFFECT ON F_B

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

As a result of this investigation, the following conclusions can be made:

1. The proposed definition for the net rock confinement pressure, taking the design of the flow cell into account, is mechanisticly superior to the one employed by Piplapure.
2. The unusual shape of the Klinkenberg plots is not only dependent upon the manner in which the flow tests have been conducted, flow test methods Case I or Case II, but it may also be affected by the limitation in the design of the present flow cell in which the rock is confined axially but not radially.
3. The effect of partial confinement of the rock sample appears to become more severe for runs on low permeability samples, i.e. LS-2 and SS-1-A, and/or runs with high mean flowing pressures, i.e. 5 and 6 on LS-1, which causes the gas to flow through the interfaces.
4. Estimated values of b and F_B are very sensitive to the number of points in the inertial region of the particular run. However, K seems to be less sensitive.
5. The suggested data discrimination methods to delineate the viscous and the inertial regions are inadequate and inconsistent. Hence,

usage of any graphical parameter estimation technique should be avoided whenever possible.

6. The inertial resistance coefficient, F_B, appears to be a property of the fluid as well as the rock.
7. The absolute permeability, K, decreases as the net rock confinement pressure increases and the effect is more pronounced in low permeability rock samples.
8. The slippage coefficient, b, and the inertial coefficient, F_B, increase as the net rock confinement pressure increases.

B. Recommendations

The following recommendations are made:

1. The present flow cell should be redesigned so as to permit radial confinement.
2. The proposed net confinement pressure definition should be modified to account for the weight of the hydraulic piston.
3. Other definitions of net confinement pressure should be examined.
4. The behavior of the rubber gasket under different stress conditions should be studied prior to its usage.
5. Flow tests with different gases should be conducted in linear systems to clarify conclusion 6.
6. The effect of confinement pressure on the parameters K, b and F_B should be studied in linear systems where both radial and axial confinement may be employed.
7. Using the differential model of the visco-inertial flow theory, as given by Equation 36, the parameters K, b and F_B should be

estimated so as to examine the effect of the assumption utilized in the derivation of the integrated model.

8. The sensitivity of the estimated parameters to the observed variables, i.e. pressure, flow rate and temperature, should be studied so as to improve the quality of experimental data.
9. Runs using flow test method Case II should be repeated and studied in detail so as to explain the nature and the shape of the resulting curves.

NOMENCLATURE

NOMENCLATURE

a	constant in Equation 60
A	area, L^2
A^*	a function of q^2 in Equation 10, dimensionless
A_o	constant in Equation 60
$A(r)$	integral defined by Equation 47
b	slippage coefficient, M/L^2t
b_o	hard-sphere molal volume, L^3
B_o,b,c	constants in Equations 60 and 61
C	a proportionality factor in Equation 3
C_A,C_B	constants in Equations 50 and 51, respectively
C_D	drag coefficient, dimensionless
C_f	Fanning friction factor, dimensionless
C_L	laminar energy loss ciefficient, dimensionless
C_T	turbulent energy loss coefficient, dimensionless
C_1,C_2	functions of Q_o, $Q_o\|Q_o\|$ and $Q_o\|Q_o^{m-1}\|$, respectively
d	a characteristic hydraulic length, L
D	denotes diameter, L
D_1,D_2	functions of Q_o, $Q_o\|Q_o\|$ and $Q_o\|Q_o^{m-1}\|$, respectively
$f()$	denotes "a function of"
F_A	viscous coefficient, $1/L^2$
F_B	inertial coefficient, $1/L$
F_C	turbulent coefficient, $Mt^{(m-3)}/L^m$

G	specific gas gravity, with G for air = 1.0
h	thickness of porous medium, L
k	Boltzman constant
K	absolute permeability, L^2
K_a	apparent permeability, L^2
k_1, k_2, k_3	proportionality constants
L	length, L
ℓ_f	lithology factor in Equation 8
m	derived turbulent intensity term, dimensionless
M	molecular weight, M
m(P)	integral term used with Equation 4
n	turbulent intensity term, dimensionless
N_{Re}	Reynolds number, dimensionless
n_1, n_2	exponents in Equation 14
P	pressure, M/Lt^2
q	superficial velocity, L/t
Q_o	volumetric flow rate, L^3/t
r	radial distance or radius, L
R	universal gas constant, ML^2/t^2T
R	coefficient of correlation defined by Equation 52, dimensionless
T	temperature, T
T*	Lennard-Jones potential parameter
t, t_o	time, t
V	volume, L^3

V_t	instantaneous turbulent or total velocity, L/t
$\bar{V}$	bulk fluid velocity, L/t
V'	local turbulent velocity fluctuation component, L/t
x	denotes distance or direction, L
W	mass flow rate, M/t
Z	compressibility factor, dimensionless

Greek

α,β	constants in Equation 5
β	parameter defined by Equation 59
γ	parameter defined by Equation 60
δ	parameter defined by Equation 61
ε	a small number as convergence criterion
ε_i	error term defined by Equation 53
ε_o	maximum energy of molecular attraction, ML^2/t^2
$\theta_1,\theta_2,\theta_3$	components of the parameter vector, $\underline{\theta}$
λ	a convergence factor in the Marquart algorithm
$\bar{\lambda}$	mean free path length, L
μ	viscosity, M/Lt
ρ	density, M/L^3
ρ^*	reduced density, dimensionless
σ	distance between plates of orifice model, L
σ_{ss}	structural support pressure, M/Lt^2
σ^*	hard-sphere diameter, L
ϕ	porosity, dimensionless
Ω_v	collision integral

Subscripts

e	reference to outer extent of porous media
hc	hydraulic confinement
i	denotes "inertial", the i^{th} point or the i^{th} iteration
m	mean
nc	net confinement
o	reference conditions
p	piston or particle
r	radial vector designator
sc	standard conditions
v	denots "viscous"
w	reference to wellbore
1	reference to inlet
2	reference to outlet

Superscripts

-	denotes an average value
o	denotes degree
'	denotes prime

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APPENDICES

APPENDIX A

DEVELOPMENT OF THE UBEROI FLOW EQUATION

APPENDIX A

In order to describe turbulent fluid flow, it is important to account for the effects of turbulent velocities. Due to the random nature of the turbulent motions, analysis and solution must be in terms of the statistical approach (69). Thus the instantaneous turbulent velocity in the direction of r, V_t can be written as (70)

$$V_t = \bar{V}_r + V'_r \qquad \text{(A-1)}$$

where $\bar{V}_r$ is the time-smoothed bulk fluid velocity, averaged over a relatively long period of time, and is given by

$$\bar{V}_r = \frac{1}{t_o} \int_t^{t+t_o} V_t dt \qquad \text{(A-2)}$$

and where V'_r is the local turbulent velocity fluctuation component.

Turbulence is defined as <u>homogeneous</u> when its scale and intensity are independent of coordinate position. It is further defined as being <u>isotropic</u> when these characteristics are independent of direction. Thus, isotropic turbulence requires local turbulence to have the mean values of flow parameters which are independent of translation, rotation, and reflection on the axis of reference as reported by Townsend (71). However, the entire flow system exhibits inhomogeneous turbulence due to varying bulk velocity. Hence, consistent with isotropic inhomogeneous turbulence, Crafton (72) proposed the following relationship for the local turbulent velocity:

$$V_r' = k_1 V_r^n , \qquad n \neq 1 \tag{A-3}$$

where n is called the "turbulent intensity term" and given by

$$n = \sqrt{\overline{V_r'^2}}/\langle \bar{V}_r \rangle \tag{A-4}$$

which is a measure of the magnitude of turbulence and angular parenthesis < > denotes an average. However, the validity of this equation A-3 is questionable since the fluctuation velocity component $V_r'(r,t)$, a function of both space and time, can not be given in terms of the fluid bulk velocity $\bar{V}_r(r)$ which is a function of space only as indicated by Equation A-2.

Equation A-3 is substantiated by Uberoi's empirical work (73) performed on free air jets. Uberoi's Equation 6 can be rewritten as

$$\frac{d\bar{V}_r}{dr} = k_2 \mu \overline{\left(\frac{\partial V_r'}{\partial r}\right)^2} \tag{A-5}$$

Multiplying both sides of this equation by $\rho \bar{V}_r$ yields

$$\rho \bar{V}_r \left(\frac{d\bar{V}_r}{dr}\right) = k_2 \mu \rho \bar{V}_r \overline{\left(\frac{\partial V_r'}{\partial r}\right)^2} \tag{A-6}$$

Differentiation of Equation A-3 with respect to r yields

$$\frac{\partial V_r'}{\partial r} = n k_1 \bar{V}_r^{(n-1)} \left(\frac{\partial \bar{V}_r}{\partial r}\right) \tag{A-7}$$

Crafton made an error in the above differentiation by not including the $\left(\frac{\partial \bar{V}_r}{\partial r}\right)$ term.

Now, note the identity

$$\rho \bar{V}_r \, d\bar{V}_r = \frac{1}{2} d(\bar{V}_r^2 \rho) \equiv \frac{1}{2} k_3 d(P) \tag{A-8}$$

and the combination of Equations A-6, A-7, and A-8 will yield

$$\frac{dP}{dr} = \frac{2\rho}{n^2 k_1^2 k_2 k_3 \mu} \bar{V}_r^{(3-2n)} \tag{A-9}$$

as compared to the same equation derived by Crafton

$$\frac{dP}{dr} = \frac{2n^2 k_1^2 k_2 \rho}{\mu} \bar{V}_r^{(2n-1)} \equiv B\rho \bar{V}_r^{(2n-1)} \tag{A-10}$$

Using the energy superposition principle with Equation A-9 to include the pressure gradient due to permeability effects, i.e. the laminar boundary layer, the complete pressure gradient will be

$$-\frac{dP}{dr} = \frac{\mu}{K} \bar{V}_r + \frac{F_C}{\mu} \rho \bar{V}_r |\bar{V}_r^{(m-1)}| \tag{A-11}$$

where

$$F_C = \frac{2}{n^2 k_1^2 k_2 k_3} \tag{A-12}$$

$$m = 3-2n \tag{A-13}$$

as opposed to the definition given by Crafton as

$$m = 2n-1 \tag{A-14}$$

and m is referred to as the "derived turbulent intensity term".

Equation A-11 may be referred to as the "Uberoi flow equation". However, the second term of this equation indicates that pressure gradient will decrease as the fluid viscosity increases which is exactly opposite to what one expects.

In view of this and the questionable nature of Equation A-3, the usage of Uberoi flow equation should be avoided until further clarification is made.

APPENDIX B

APPLICATION OF THE NUMERICAL AND THE SEMI-GRAPHICAL PARAMETER ESTIMATION TECHNIQUES TO THE INTEGRATED MODEL OF VISCO-INERTIAL FLOW THEORY

APPENDIX B

PART I. Application of the Gauss-Newton Method as a Non-Linear Least Squares Parameter Estimation Technique

The general integrated 3-parameter model, non-linear with respect to the parameters, given by Equation 48 can be rewritten as

$$P_e = \left[P_w^2 + \frac{1424\bar{\mu}\bar{Z}\bar{T}Q_o \ln(r_e/r_w)}{hK} - 2b(P_e - P_w) + \frac{3.1602 \times 10^{-12} F_B \bar{G}\bar{T}\bar{Z}Q_o^2}{h^2}\left(\frac{1}{r_w} - \frac{1}{r_e}\right)\right]^{\frac{1}{2}} \quad \text{(B-1)}$$

with parameters K, b and F_B to be estimated.

Equation B-1 can be expressed in a general mathematical form as

$$P_{e,i} = f(Q_{o,i}, \underline{\theta}) + E_i \quad \text{(B-2)}$$

or

$$\hat{P}_{e,i} = f(Q_{o,i}, \underline{\theta}) \quad \text{(B-3)}$$

where

$P_{e,i}$ = experimental value of the dependent variable for the i^{th} observation

$\hat{P}_{e,i}$ = predicted or calculated value of the dependent variable for the i^{th} observation

$E_i = P_{e,i} - \hat{P}_{e,i}$ = residual term representing the error between $P_{e,i}$ and $\hat{P}_{e,i}$ for the i^{th} observation

$Q_{o,i}$ = measured value of the independent variable for the i^{th} observation

$\underline{\theta}$ = $(\theta_1, \theta_2, \theta_3) = (K\ b\ F_B)^T$ is a vector of the parameters in the model

f = functional relationship known as the response function

The Gauss-Newton procedure is based on a linearization of the proposed Equation B-2. A least squares objective function is utilized and the method has proved effective when good starting estimates of the unknown parameters are available. The algorithm proceeds as follows:

1. Linearize Equation B-3 by expanding $\hat{P}_{e,i}$ in a Taylor series about current trial values for the parameters and retaining the linear terms only, as

$$\hat{P}^j_{e,i} = \hat{P}^{j-1}_{e,i} + \left(\frac{\partial P_e}{\partial K}\right)^{j-1}_i (K^j - K^{j-1}) + \left(\frac{\partial P_e}{\partial b}\right)^{j-1}_i (b^j - b^{j-1}) + \left(\frac{\partial P_e}{\partial F_B}\right)^{j-1}_i (F^j_B - F^{j-1}_B) \qquad \text{(B-4)}$$

where

$$\left(\frac{\partial \hat{P}_e}{\partial K}\right)^{j-1}_i = -\left[\frac{712\bar{\mu}\bar{Z}\bar{T}Q_o \ln(r_e/r_w)}{K^2 h \hat{P}_e}\right]^{j-1}_i \qquad \text{(B-5)}$$

$$\left(\frac{\partial \hat{P}_e}{\partial b}\right)^{j-1}_i = -\left(\frac{P_e - P_w}{\hat{P}_e}\right)^{j-1}_i \qquad \text{(B-6)}$$

$$\left(\frac{\partial \hat{P}_e}{\partial F_B}\right)_i^{j-1} = \left[\frac{1.5801 \times 10^{-12}\ G\mu\bar{Z}Q_o^2}{h^2 \hat{P}_e}\left(\frac{1}{r_w} - \frac{1}{r_e}\right)\right]_i^{j-1} \qquad \text{(B-7)}$$

j = the j^{th} iteration and $j = 1, 2, \ldots$ ITER

2. If the number of experimental points greater than the number of unknown parameters, as in the case of this study, formulate a least squares objective function δ as

$$\delta = \sum_{i=1}^{ND} (E_i^j)^2 = \sum_{i=1}^{ND} (P_{e,i} - \hat{P}_{e,i}^j)^2 \qquad \text{(B-8)}$$

where

ND = number of experimental data points

E_i^j = random error associated with i^{th} data point and j^{th} iteration

3. Substitute the linearized model into the objective function and form the "normal equations" by setting the partial derivatives of the objective function, with respect to each parameter, equal to zero, i.e. minimize sum of the squared errors

$$\frac{\partial \delta}{\partial \theta_j} = 0, \qquad j = 1, 2, 3 \qquad \text{(B-9)}$$

The resulting normal equations will be of the form

$$(\underline{D}^T \underline{D})\ \underline{S} = \underline{D}^T \underline{E} \qquad \text{(B-10)}$$

where

$$\underline{D} = \begin{bmatrix} \left(\frac{\partial \hat{P}_e}{\partial K}\right)_1^{j-1} & \left(\frac{\partial \hat{P}_e}{\partial b}\right)_1^{j-1} & \left(\frac{\partial \hat{P}_e}{\partial F_B}\right)_1^{j-1} \\ \vdots & \vdots & \vdots \\ \left(\frac{\partial \hat{P}_e}{\partial K}\right)_{ND}^{j-1} & \left(\frac{\partial \hat{P}_e}{\partial b}\right)_{ND}^{j-1} & \left(\frac{\partial \hat{P}_e}{\partial F_B}\right)_{ND}^{j-1} \end{bmatrix}$$ Design Matrix

$$\underline{E} = \begin{bmatrix} (P_{e,1} - \hat{P}^j_{e,1}) \\ \vdots \\ (P_{e,ND} - \hat{P}^j_{e,ND}) \end{bmatrix}$$ Error Vector

and

$$\underline{S} = \begin{bmatrix} (K^j - K^{j-1}) \\ (b^j - b^{j-1}) \\ (F^j_B - b^{j-1}) \end{bmatrix}$$

$\underline{D}^T$ is the transpose of the design or sensitivity matrix $\underline{D}$ and the derivatives in $\underline{D}$ may be evaluated analytically or numerically.

4. Solve the set of normal equations, which is a system of linear algebraic equations, for $\underline{S}$ as

$$\underline{S} = \underline{A}^{-1}\,\underline{C} \qquad \text{(B-11)}$$

where

$$\underline{A} = \underline{D}^T\,\underline{D}$$

$$\underline{C} = \underline{D}^T\,\underline{E}$$

5. Compare $\left(\dfrac{|\underline{S}|}{\underline{\theta} + 0.001}\right)$ to ε

(i) if smaller than ε, exit

(ii) if larger than ε, form a new set of estimates using

$$\underline{\theta}^j = \underline{\theta}^{j-1} + \underline{S} \qquad \text{(B-12)}$$

and the process is repeated until convergence is achieved.

6. Form the variance-covariance matrix of the parameters, $\sigma^2\,\underline{A}$. Hence σ^2 is the error variance of the measurements and can be approximated by

$$\sigma^2 = \frac{\sum_{i=1}^{ND} E_i^2}{ND-NP} \qquad \text{(B-13)}$$

where

NP = number of unknown parameters, NP = 3

Consequently, the variance-covariance matrix is

$$\begin{bmatrix} \sigma_K^2 & \text{cov}(K,b) & \text{cov}(K,F_B) \\ & \sigma_b^2 & \text{cov}(b,F_B) \\ & & \sigma_{F_B}^2 \end{bmatrix}$$

and the correlation matrix will be

$$\begin{bmatrix} 1 & \dfrac{\text{cov}(K,b)}{\sigma_K \sigma_b} & \dfrac{\text{cov}(K,F_B)}{\sigma_K \sigma_{F_B}} \\ & 1 & \dfrac{\text{cov}(b,F_B)}{\sigma_b \sigma_{F_B}} \\ & & 1 \end{bmatrix}$$

These are both symmetric 3 by 3 matrices.

7. Calculate the percent error using

$$100\, E_i \equiv 100(P_{e,i} - \hat{P}_{e,i})$$

The salient points of the FORTRAN computer program, written for non-linear least squares parameter estimation using the Gauss-Newton algorithm, and presented in the following pages, may be expressed as

1. The main program is THE GAUSS-NEWTON with subroutines

   FLUID
   DESGN
   SHEN1
   INVRT
   SHEN2
   and SHEN3

2. The required input data for the main program are
   - number of different data sets, ISET
   - rock sample dimensions, r_e, r_w, and h
   - specific gas gravity, G
   - integers indicating the type of gas and the maximum number of iterations, ICONT and ITMAX
   - number of unknown parameters, NP
   - number of points in each data set, ND
   - initial guesses for the parameters, (PAR(I), I = 1, .., NP)
   - and the experimental flow data, P_e, P_w, $\bar{T}$, Q_o

Furthermore, the program can easily be modified to estimate any number of parameters by making appropriate changes in subroutine DESGN to include the partial derivatives with respect to additional parameters.

THE GAUSS-NEWTON MAIN

```
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                     *
C        INTEGRATED 3-PARAMETER MATHEMATICAL MODEL                    *
C      DESCRIBING STEADY-RADIAL VISCO-INERTIAL GAS FLOW               *
C                  THROUGH POROUS MEDIA                               *
C                                                                     *
C     TAKING THE PROCESSED FLOW DATA AS INPUT THE PROGRAM             *
C     GIVES THE LEAST SQUARES ESTIMATES OF THE UNKNOWN                *
C          PARAMETERS,I.E.(PAR(I),I=1,2,....NP)                       *
C                          USING                                      *
C     THE GAUSS-NEWTON METHOD,A NON-LINEAR LEAST SQUARES              *
C     ALGORITHM,AS A PARAMETER ESTIMATION TECHNIQUE                   *
C                                                                     *
C     NOMENCLATURE                                                    *
C     ISET=NUMBER OF DATA SET                                         *
C     RE = OUTER RADIUS OF THE ROCK SAMPLE,IN FT.                     *
C     RW = BORE HOLE RADIUS OF THE ROCK SAMPLE,IN FT.                 *
C      H = THICKNESS OF THE ROCK SAMPLE,IN FT.                        *
C      G = SPECIFIC GRAVITY OF THE FLOWING FLUID(GAS)                 *
C     ICONT=A COUNTER SET TO DIFERENTIATE THE GASES USED              *
C          IN CONJUNCTION TO SUBROUTINE 'FLUID'                       *
C     ICONT=1,FOR NITROGEN                                            *
C          =2,FOR ARGON                                               *
C          =3,FOR METHANE                                             *
C     ITMAX=SEE SUBROUTINE 'FLUID'                                    *
C     NP = NUMBER OF UNKNOWN PARAMETERS                               *
C     ND = NUMBER OF POINTS IN EACH DATA SET                          *
C                                                                     *
C     INITIAL GUESSES FOR THE UNKNOWN PARAMETERS                      *
C     PAR(1)= AN ESTIMATE FOR PERMEABILITY K,                         *
C             IN MILLIDARCIES                                         *
C     PAR(2)= AN ESTIMATE FOR SLIPPAGE COEFFICIENT B,                 *
C             IN PSIA                                                 *
C     PAR(3)= AN ESTIMATE FOR INERTIAL RESISTANCE                     *
C             COEFFICIENT FB,IN 1/FT.                                 *
C     PE = PRESSURE AT THE OUTER RADIUS,IN PSIA                       *
C     PW = PRESSURE AT THE BORE HOLE,IN PSIA                          *
C      T = ARITHMETIC MEAN TEMPERATURE ,IN DEGRES F.                  *
C     QQ = FLOW RATE,IN MSCF/DAY                                      *
C     MU = VISCOCITY OF THE FLOWING FLUID EVALUATED AT                *
C          THE ARITHMETIC MEAN PRESSURE AND TEMPERATURE,              *
C          IN CENTIPOISE                                              *
C     MUTZ= MU*T*Z                                                    *
C     WHERE                                                           *
C         Z = COMPRESSIBILITY OF THE GAS EVALUATED AT THE             *
C             ARITHMETIC MEAN PRESSURE AND TEMPERATURE                *
C      ITER = NUMBER OF ITERATIONS FOR CONVERGENCE                    *
C                                                                     *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
```

THE GAUSS-NEWTON MAIN ...(CONT'D)

```
      REAL MU(40),MUTZ(40)
      DIMENSION A(3,3),S(3),SR(3),C(40),ERR(40),E(40),
     1CORR(3,3),COV(3,3),D(40,3),P(40),T(40),Z(40),
     2HEAD(7,12),TOP1(12),TOP2(12)
      COMMON PE(40),PW(40),QO(40),F(40),PAR(3),MU,MUTZ
      READ(5,15)ISET
      DO 1 MM =1,ISET
      READ(5,5)RE,RW,H
    5 FORMAT(3F10.5)
      READ(5,10)G
   10 FORMAT(F10.5)
      READ(5,11)ICONT,ITMAX
   11 FORMAT(2I5)
      READ(5,15)NP
      READ(5,15)ND
   15 FORMAT(I5)
      READ(5,20)(PAR(I),I=1,NP)
   20 FORMAT(3E11.4)
      READ(5,22)((HEAD(I,J),I=1,7),J=1,12)
   22 FORMAT(12A4)
      READ(5,23)(TOP1(I),I=1,12)
      READ(5,23)(TOP2(I),I=1,12)
   23 FORMAT(12A4)
      WRITE(6,25)((HEAD(I,J),I=1,7),J=1,12)
   25 FORMAT('1',/////36X,12A4///29X,12A4//24X,12A4/24X,
     112A4/24X,12A4/24X,12A4/24X,12A4/)
      READ(5,26)(PE(I),PW(I),T(I),QO(I),I=1,ND)
   26 FORMAT(3F11.6,E13.6)
      WRITE(6,27)
   27 FORMAT(9X,'P(EXT)        P(WELL)        FLOW RATE',
     1'     VISCOSITY        MU*T*Z',/9X,
     2'(PSIA)         (PSIA)         (MSCF/D)          (CP)',
     3'         (CP*DEG R)',/)
      DO 30 I=1,ND
      P(I)=(PE(I)+PW(I))/2.
      CALL FLUID(I,ICONT,G,ITMAX,P,T,TA,MU,Z)
   30 MUTZ(I)=MU(I)*TA*Z(I)
      DO 33 I=1,ND
   33 WRITE(6,35)PE(I),PW(I),QO(I),MU(I),MUTZ(I)
   35 FORMAT(' ',5X,5(E13.6,1X))
      WRITE(6,40)(TOP1(I),I=1,12)
   40 FORMAT('1',//////42X,12A4)
      WRITE(6,44)
   44 FORMAT(//22X,'   NUMERICAL PARAMETER ESTIMATION'
     1' TECHNIQUE'//33X,'THE GAUSS-NEWTON METHOD'/24X,
     2'I.E. A NON-LINEAR LEAST SQUARES ALGORITHM'//38X,
     3'LINEAR SQUARES ESTIMATES')
```

THE GAUSS-NEWTON MAIN ...(CONT'D)

```
      WRITE(6,77)
   77 FORMAT(/23X,'ITER',10X,'K',11X,'B',11X,'FB')
      WRITE(6,78)
   78 FORMAT(24X,'NO',7X,'  (MD)          (PSIA)          '
     1'(1/FT)')

C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                   *
C     READS INITIAL GUESSES TO PARAMETRS,THUS STARTS THE            *
C     ITERATION PROCEDURE AND CONTINUES UNTIL THE                   *
C     PREDEFINED CONVERGENCE CRITERION ACHIEVED                     *
C                  I.E.,EPS=0.00001                                 *
C                                                                   *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C

      EPS=0.00001
      WRITE(6,76)(PAR(I),I=1,NP)
   76 FORMAT('0',19X,'IN. GUESS',2X,E11.4,2(2X,E11.4))
      ITER=1
      A1=ALOG(RE/RW)
      A2=(1/RW-1/RE)
  100 CALL DESGN(ND,A1,A2,H,G,D,E)
      CALL SHEN1(ND,NP,D,A)
      CALL INVRT(A,NP,DET,KK)
      IF(KK-2)110,45,45
   45 CALL SHEN2(ND,NP,D,E,C)
      DO 50 I=1,NP
      CALL SHEN3(NP,A,C,S)
      SR(I)=ABS(S(I))/(0.001+ABS(PAR(I)))
      IF(SR(I)-EPS)50,50,60
   50 CONTINUE
      DO 70 I=1,NP
   70 PAR(I)=PAR(I)+S(I)
      SUM=0.0
      DO 80 I=1,ND
      ERR(I)=E(I)/PE(I)*100.
   80 SUM=SUM+E(I)**2
      WRITE(6,85) ITER,(PAR(I),I=1,NP)
   85 FORMAT(21X,I4,6X,E11.4,2(2X,E11.4))

C     CALCULATE POROSITY USING THE ESTIMATES FOR K AND FB *

      C2=ALOG(PAR(2))
      C3=ALOG(PAR(3))
      POR=EXP((20.44-C3-1.12*C2)/1.67)
      WRITE(6,88)POR
   88 FORMAT(/20X,'FRACTIONAL POROSITY =',E11.4)
```

THE GAUSS-NEWTON MAIN ...(CONT'D)

```
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                     *
C     STATISTICAL ANALYSES,I.E. SUM OF THE SQUARES OF                 *
C     ERRORS,VARIANCE,VARIANCE-COVARIANCE AND CORRELATION             *
C                          MATRICES                                   *
C                                                                     *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

      FR=ND-NP
      VAR=SUM/FR
      SDEV=SQRT(VAR)
      WRITE(6,90)SUM,VAR,SDEV
   90 FORMAT(//20X,'SUM OF ERROR SQUARES =',E11.4/,20X,
     1'VARIANCE                =',E11.4/,20X,'STANDARD'
     2,1X,'DEVIATION   =',E11.4)
      DO 95 I=1,NP
      DO 95 J=1,NP
      COV(I,J)=VAR*A(I,J)
   95 CORR(I,J)=A(I,J)/SQRT(A(I,I)*A(J,J))
      WRITE(6,113)
  113 FORMAT(//20X,'VARIANCE-COVARIANCE MATRIX')
      DO 115 I=1,NP
  115 WRITE (6,112) (COV(I,J),J=1,NP)
  112 FORMAT(/17X,4(2X,E13.5))
      WRITE(6,114)
  114 FORMAT(//20X,'CORRELATION MATRIX')
      DO 101 I=1,NP
  101 WRITE(6,111) (CORR(I,J),J=1,NP)
  111 FORMAT(/13X,4(2X,F13.5))

C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                     *
C     CALCULATIONS OF THE RELATIVE PERCENTAGE ERROR                   *
C     BETWEEN THE OBSERVED FUNCTION VALUE AND THAT                    *
C     CALCULATED USING THE PARAMETERS OBTAINED                        *
C     FROM NON-LINEAR LEAST SQUARES ANALYSIS                          *
C                                                                     *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

      WRITE(6,107)(TOP2(I),I=1,12)
  107 FORMAT('1',//////32X,12A4)
      WRITE(6,108)
  108 FORMAT(//12X,'OBS. FUNCTION       CALC. FUNCTION'
     1,4X,'ERROR (PERCENT)'//)
      DO 119 I=1,ND
  119 WRITE(6,120)  PE(I),F(I),ERR(I)
  120 FORMAT(' ',4X,3(4X,E15.5))
      GO TO 1
```

THE GAUSS-NEWTON MAIN ...(CONT'D)

```
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                   *
C     COMPUTATION OF UPDATED PARAMETERS AND A CHECK FOR             *
C           NO. OF ITERATION AND CONVERGENCE                        *
C                                                                   *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

   60 DO 125 I=1,NP
  125 PAR(I)=PAR(I)+S(I)
      SUM=0.0
      DO 130 I=1,ND
  130 SUM=SUM+E(I)**2
      WRITE(6,85) ITER,(PAR(I),I=1,NP)
      IF(ITER-30)140,140,150
  140 ITER=ITER+1
      GO TO 100
  110 WRITE(6,160)
  160 FORMAT(//20X,'SINGULAR MATRIX')
      GO TO 1
  150 WRITE(6,170)
  170 FORMAT(//20X,'NO CONVERGENCE')
    1 CONTINUE
      CALL EXIT
      END
```

```
      SUBROUTINE  FLUID

C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                     *
C     IT EVALUATES THE FLUID PROPERTIES,I.E. VISCOSITY                *
C     AND COMPRESSIBILITY,OF THE GASES USED AS FLOWING                *
C               FLUIDS IN THIS STUDY                                  *
C                                                                     *
C     ICONT=1,NITROGEN                                                *
C          =2,FOR ARGON                                               *
C          =3,FOR METHANE                                             *
C     ITMAX= NO. OF ITERATION SET IN CALCULATING THE                  *
C            MOLAL VOLUME OF METHANE,I.E. Z - FACTORS                 *
C                                                                     *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      SUBROUTINE FLUID(I,ICONT,G,ITMAX,P,T,TA,MU,Z)

      REAL MU(40),MUO
      DIMENSION AA(5),BB(3),XX(5),ZZ(5),C(5),P(40),T(40),
     1Z(40)
      DATA AA/2.2769,0.05587,0.01855,-0.01587,128300./
      DATA BB/148.6,3.758,66.98/
      DATA XX/504.,522.,540.,558.,576./
      PATM=P(I)/14.696
      TA=T(I)+459.7
      TK=TA/1.8
      GO TO(11,22,33),ICONT

C     ........   FOR  NITROGEN   ......

   11 ZZ(1)=.999986E 00-.370971E-03*PATM+.211789E-05*PATM
     1**2+.702961E-08*PATM**3
      ZZ(2)=.999985E 00-.268006E-03*PATM+.211419E-05*PATM
     1**2+.524241E-08*PATM**3
      ZZ(3)=.100000E 01-.183813E-03*PATM+.211419E-05*PATM
     1**2+.263740E-08*PATM**3
      ZZ(4)=.999667E 00-.388886E-03*PATM+.988604E-05*PATM
     1**2+.488241E-07*PATM**3
      ZZ(5)=.100000E 01-.405702E-04*PATM+.198236E-05*PATM
     1**2+.325175E-09*PATM**3
      MU(I)=100.0*(.1778E-03*(1.+.8958E-03*(PATM-1.)+.612E
     *-06
     1*(PATM-1.)**2+.3997E-07*(PATM-1.)**3)+.455E-06*
     2(((T(I)-32.)/1.8)-25.))
      GO TO 44
```

SUBROUTINE FLUID ...(CONT'D)

```
C     ........   FOR  ARGON   .......

   22 ZZ(1)=.100004E+01-.881571E-03*PATM+.180764E-05*PATM
     1**2+.403805E-08*PATM**3
      ZZ(2)=.100003E+01-.752411E-03*PATM+.173597E-05*PATM
     1**2+.325023E-08*PATM**3
      ZZ(3)=.100001E+01-.635479E-05*PATM+.156902E-05*PATM
     1**2+.314538E-08*PATM**3
      ZZ(4)=.100003E+01-.543677E-03*PATM+.164908E-05*PATM
     1**2+.154703E-08*PATM**3
      ZZ(5)=.100003E+01-.458263E-03*PATM+.161105E-05*PATM
     1**2+.684521E-09*PATM**3
      MU(I)=100.*(.2262E-03*(1.+.8945E-03*(PATM-1.)+.4930E
     *-05
     1*(PATM-1.)**2-.7200E-07*(PATM-1.)**3)+.6370E-06*
     2(((T(I)-32.)/1.8)-25.))
   44 Z(I)=0.
      DO 20 K=1,5
      C(K)=1.
      DO 25 J=1,5
      IF(J-K)15,25,15
   15 C(K)=C(K)*((TA-XX(J))/(XX(K)-XX(J)))
   25 CONTINUE
   20 Z(I)=Z(I)+C(K)*ZZ(K)
      GO TO 10

C     ........  FOR METHANE  .......
C     ***********************************************************
C     *                                                         *
C     .... THE MOLAL VOLUME AND Z - FACTOR CALCULATION ..
C     .... BY THE BEATTIE-BRIDGMAN EQUOTION OF STATE   ..
C     *                                                         *
C     ***********************************************************
C     .... COMPUTE TEMPERATURE-DEPENDENT PARAMETERS ....

   33 R=0.08205
      BETA=R*TK*AA(2)-AA(1)-R*AA(5)/(TK*TK)
      GAMMA=-R*TK*AA(2)*AA(4)+AA(1)*AA(3)-R*AA(5)*AA(2)/
     1(TK*TK)
      DELTA=R*AA(2)*AA(4)*AA(5)/(TK*TK)

C     .... USE IDEAL GAS LAW FOR FIRST VOLUME ESTIMATE ....

      V=R*TK/PATM

C     .... BEGIN NEWTON METOD ITERATION ....

      DO 30 ITER=1,ITMAX
```

SUBROUTINE FLUID ...(CONT'D)

```
      DELV=(((((-PATM*V+R*TK)*V+BETA)*V+GAMMA)*V+DELTA)*V)/
     1(((R*TK*V+2.*BETA)*V+3.*GAMMA)*V+4.*DELTA)
      V=V+DELV

C     .... CHECK FOR CONVERGENCE ....

      IF(ABS(DELV/V)-0.00001)40,40,30
   40 Z(I)=PATM*V/(R*TK)
      WM=28.97*G
      TD=TK/BB(1)
      RV=0.697*(1.+0.323*ALOG(TD))
      MUO=0.002669*SQRT(WM*TK)*RV/BB(2)**2
      GD=PATM/(Z(I)*R*TK*1000.)
      RD=BB(3)*GD
      MU(I)=MUO*(1.+(((0.61*RD+0.96)*RD+0.55)*RD)/(TK*
     **0.59))
      GO TO 10
   30 CONTINUE
      WRITE(6,50)
   50 FORMAT('0',20X,'NO CONVERGENCE AT ITMAX')
   10 RETURN
      END
```

SUBROUTINE DESGN

```
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                     *
C     IT CALCULATES THE  ND BY NP DESIGN MATRIX D USING               *
C     THE PARTIAL DERIVATIVES OF THE FUNCTION F(I) WITH               *
C         RESPECT TO UNKNOWN PARAMETERS K,B,AND FB                    *
C                                                                     *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      SUBROUTINE DESGN(ND,A1,A2,H,G,D,E)

      REAL MU(40),MUTZ(40)
      DIMENSION D(40,3),E(40)
      COMMON PE(40),PW(40),QO(40),F(40),PAR(3),MU,MUTZ
      DO 10 I=1,ND
      F(I)=(PW(I)**2+1424.*MUTZ(I)*QO(I)*A1/(H*PAR(1))
     1-2.*PAR(2)*(PE(I)-PW(I))+0.31602E-11*PAR(3)*G*MUTZ(I)
     2*QO(I)*QO(I)*A2/(H*H*MU(I)))**0.5
      D(I,1)=-712.*MUTZ(I)*QO(I)*A1/(H*PAR(1)*PAR(1)*F(I))
      D(I,2)=-(PE(I)-PW(I))/F(I)
      D(I,3)=0.15801E-11*G*MUTZ(I)*QO(I)*QO(I)*A2/(H*H*MU(I)
     **
     1F(I))
   10 E(I)=PE(I)-F(I)
      RETURN
      END
```

```
      SUBROUTINE  SHEN1

C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                 *
C          IT FORMS THE  NP BY NP  MATRIX A THROUGH              *
C     POSTMULTIPLYING DESIGN MATRIX D BY ITS TRANSPOSE           *
C                                                                 *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      SUBROUTINE SHEN1(ND,NP,D,A)

      DIMENSION D(40,3),A(3,3)
      DO 10 I=1,NP
      DO 10 K=I,NP
      A(I,K)=0.
      DO 10 J=1,ND
   10 A(I,K)=A(I,K)+D(J,K)*D(J,I)
      LL=NP-1
      DO 20 I=1,LL
      II=I+1
      DO 20 J=II,NP
   20 A(J,I)=A(I,J)
      RETURN
      END
```

```
      SUBROUTINE  INVRT

C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                     *
C     IT INVERTS THE M BY M SYMMETRIC, POSITIVE DEFINITE              *
C     MATRIX A AND STORES THE RESULTANT INVERSE UNDER THE             *
C                        SAME NAME                                    *
C        M = NO. OF PARAMETERS IN THE ERROR MODEL,I.E. NP             *
C       DET= THE DETERMINANT OF MATRIX A                              *
C     IF KK=1, MATRIX A CANNOT BE INVERTED AND                        *
C              COMPLETED                                              *
C     IF KK=2, INVERSION IS POSSIBLE,AND HAS ALREADY                  *
C              BEEN COPLETED                                          *
C                                                                     *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      SUBROUTINE INVRT(A,NP,DET,KK)

      DIMENSION A(3,3),B(3,3),R(3,3),S(3,3),TOL(3)

C     PHASE 1. COMPUTE THE UPPER TRIANGULAR MATRIX,R,WHICH
C              WILL GIVE MATRIX A WHEN MULTIPLIED BY ITS
C                        TRANSPOSE
C     THE DIAGONAL ELEMENTS OF R SHOULD BE GREATER THAN
C     ZERO. THE TOLERANCE FOR THE J TH. DIAGONAL ELEMENT,
C     DENOTED BY TOL(J),IS ARBITRARILY SET EQUAL TO THE
C     J TH. DIAGONAL ELEMENT OF MATRIX A TIMES 1.0E-8.
C
      EPS=1.0E-8
      TOL(1)=EPS
      IF (A(1,1)-TOL(1))    18,18,12
   12 R(1,1)=SQRT(A(1,1))
      DO 10 K=2,NP
   10 R(1,K)=A(1,K)/R(1,1)
      DO 20 J=2,NP
      JJ=J-1
      TOL(J)=ABS(EPS*A(J,J))
      DO 20 K=J,NP
      S(J,K)=0.
      DO 30 I=1,JJ
   30 S(J,K)=S(J,K)+R(I,J)*R(I,K)
      B(J,K)=A(J,K)-S(J,K)
      IF (J-K) 15,17,15
   17 IF (B(J,K)-TOL(J)) 18,18,19
   19 R(J,K)=SQRT(B(J,K))
      GO TO 20
   15 R(J,K)=B(J,K)/R(J,J)
   20 CONTINUE
```

SUBROUTINE INVRT ...(CONT'D)

```
C       PHASE 1 COMPLETED.
C
C       COMPUTE THE DETERMINANT

        DET=1.
        DO 40 I=1,NP
        DET=DET*R(I,I)
   40   B(I,I)=1./R(I,I)
        DET=DET**2

C       PHASE 2. COMPUTE THE INVERSE OF A BY POSTMULTIPLYING
C                THE INVERSE OF R BY ITS TRANSPOSE

        MM=NP-1
        DO 60 L=1,MM
        I=NP-L
        II=I+1
        DO 60 K=II,NP
        S(I,K)=0.
        DO 80 J=II,K
   80   S(I,K)=S(I,K)+R(I,J)*B(J,K)
   60   B(I,K)=-S(I,K)/R(I,I)
        DO 95 I=1,NP
        DO 95 K=I,NP
        A(I,K)=0.
        DO 95 J=K,NP
   95   A(I,K)=A(I,K)+B(I,J)*B(K,J)
        DO 96 I=1,MM
        II=I+1
        DO 96 J=II,NP
   96   A(J,I)=A(I,J)
        KK=2
        GO TO 90
   18   KK=1
   90   RETURN
        END
```

```
      SUBROUTINE  SHEN2

C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                   *
C     IT CALCULATES THE VECTOR  C(NP)  BY POSTMULTIPLYING           *
C     ERROR VECTOR E(ND) BY THE TRANSPOSE OF  D(ND,NP)              *
C                                                                   *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      SUBROUTINE SHEN2(ND,NP,D,E,C)

      DIMENSION D(40,3),E(40),C(3)
      DO 10 I=1,NP
      C(I)=0.
      DO 10 J=1,ND
   10 C(I)=C(I)+D(J,I)*E(J)
      RETURN
      END
```

SUBROUTINE SHEN3

```
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C                                                                   *
C     IT CALCULATES THE VECTOR  S(NP)  WHICH GIVES THE              *
C     DIFFERENCE BETWEEN THE PARAMETERS AS ESTIMATED BY             *
C                TWO CONSECUTIVE ITERATION .                        *
C                                                                   *
C     * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
      SUBROUTINE SHEN3(NP,A,C,S)

      DIMENSION A(3,3),C(3),S(3)
      DO 10 I=1,NP
      S(I)=0.
      DO 10 J=1,NP
   10 S(I)=S(I)+A(I,J)*C(J)
      RETURN
      END
```

PART II. Salient Points of the Proposed Semi-Graphical Parameter Estimation Technique

The FORTRAN computer program DISCRIMINATION, presented in the following pages, performs a simultaneous linear least squares fit to the Klinkenberg and the Modified Visco-Inertial Plots to estimate the values of K, b, and F_B by using the three proposed data discrimination methods.

The subroutine programs to be used with DISCRIMINATION are LFIT and METOD.

The input data required by DISCRIMINATION are as follows:

- number of different data sets, ISET
- rock sample dimensions, r_e, r_w, and h
- specific gas gravity, G
- starting number of points, NS
- number of points in each data set, ND
- and the processed experimental flow data,

  P_e, P_w, MUTZ, MU, Q_o

  where

  $MUTZ = \bar{\mu} \cdot \bar{T} \cdot \bar{Z}$

DISCRIMINATION MAIN

```
C     ************************************************************
C                                                                  *
C     A SEMI-GRAPHICAL PARAMETER ESTIMATION TECHNIQUE              *
C                                                                  *
C     IT IS A SIMULTANEOUS LINEAR LEAST SQUARES FIT TO             *
C     KLINKENBERG AND MODIFIED VISCO-INERTIAL PLOTS TO             *
C        OBTAIN THE VALUES OF PARAMETERS K,B,AND FB                *
C                          BY                                      *
C     USING THE PROPOSED DATA DISCRIMINATION METHODS               *
C     GIVEN AS                                                     *
C                                                                  *
C           METHOD 1. TO FIND THE BEST FIT FOR THE                 *
C                     KLINKENBERG PLOT,I.E. R(MAX)                 *
C                                                                  *
C           METHOD 2. TO MATCH THE VALUES OF KV AND KI             *
C                     I.E. TO FIND THE MINIMUM RELATIVE            *
C                     ERROR,ERRK,BETWEEN KV AND KI                 *
C                                                                  *
C           METHOD 3. TO ESTIMATE THE REYNOLDS NUMBER              *
C                     I.E. IF REYNOLDS NUMBER,NRE,IS               *
C                     LESS THAN 0.01 THEN THE POINTS               *
C                     ARE IN VISCOUS REGION                        *
C                                                                  *
C     NOMENCLATURE                                                 *
C     NS = STARTING NO. OF POINTS FOR THE LINEAR LEAST             *
C          SQUARES FIT OF THE KLINKENBERG PLOT                     *
C     NV = NUMBER OF DATA POINTS IN VISCOUS REGION                 *
C     NI = NUMBER OF DATA POINTS IN INERTIAL REGION                *
C     RV = THE COEFFICIENT OF VARIATIN FOR THE FIT TO              *
C          THE KLINKENBERG PLOT                                    *
C     RI = THE COEFFICIENT OF VARIATIN FOR THE FIT TO              *
C          THE MODIFIED VISCO-INERTIAL PLOT                        *
C     INEFF = INERTIAL EFFECTS                                     *
C     VSEFF = VISCOUS EFFECTS                                      *
C     SLEFF = SLIPPAGE EFFECTS                                     *
C       NRE = THE REYNOLD NUMBER                                   *
C      ERRK = THE RELATIVE ERROR BETWEEN KV AND KI                 *
C     KV = ESTIMATE OF THE PERMEABILITY FROM THE                   *
C          KLINKENBERG PLOT,IN MILLIDARCIES                        *
C      B = ESTIMATE OF THE SLIPPAGE COEFFICIENT FROM THE           *
C          KLINKENBERG PLOT,IN PSIA                                *
C     KI = ESTIMATE OF THE PERMEABILITY FROM THE MODIFIED          *
C          VISCO-INERTIAL PLOT,IN MILLIDARCIES                     *
C     FB = ESTIMATE OF THE INERTIAL COEFFICIENT,IN (1/FT.)         *
C     NOTE ... SEE THE COMPUTER PRINT OUT PRESENTED FOR            *
C              'THE GAUSS-NEWTON METHOD' FOR ADDITIONAL            *
C                        NOMENCLATURE REQUIRED                     *
C                                                                  *
C     ************************************************************
```

DISCRIMINATION MAIN ...(CONT'D)

```
      REAL MU(40),MUTZ(40),KV(40),KI(40),INEFF,NRE(40)
      DIMENSION PE(40),PW(40),QQ(40),HEAD(12),XV(40),
     1YV(40),B(40),S2V(40),RV(40),XI(40),YI(40),YIM(40),
     2XIM(40),FB(40),S2I(40),RI(40),ERRK(40)
      READ(5,15)ISET
      DO 1 MM=1,ISET
      READ(5,5)RE,RW,H
    5 FORMAT(3F10.5)
      READ(5,10)G
   10 FORMAT(F10.5)
      READ(5,15)NS
      READ(5,15)ND
   15 FORMAT(I5)
      READ(5,20)(HEAD(I),I=1,12)
   20 FORMAT(12A4)
      WRITE(6,25)(HEAD(I),I=1,12)
   25 FORMAT('1',/////35X,12A4)
      READ(5,30)(PE(I),PW(I),MUTZ(I),MU(I),QQ(I),I=1,ND)
   30 FORMAT(5E13.6)

C     ..... KLINKENBERG PLOT ....

      CA=1424.*ALOG(RE/RW)/H
      DO 40 I=1,ND
      YV(I)=CA*MUTZ(I)*QQ(I)/(PE(I)**2-PW(I)**2)
   40 XV(I)=2./(PE(I)+PW(I))
      NF=ND-3
      DO 44 NV=NS,NF
      CALL LFIT(NV,YV,XV,A1,A0,S2,R)
      KV(NV)=A0
      B(NV)=A1/A0
      S2V(NV)=S2
   44 RV(NV)=R
      WRITE(6,45)
   45 FORMAT(//18X,'A SEMI-GRAPHICAL METHOD OF '
     1'PARAMETER ESTIMATION'/14X,'I.E. A SIMULTANEOUS '
     2'LINEAR LEAST SQUARES FIT TECHNIQUE'//10X,
     3'NV     B(NV)        KV(NV)       KI(NI)       FB(NI)'
     4'     NRE(NV)')

C     .... MODIFIED VISCO-INERTIAL PLOT ....

      CB=0.31602E-11*G*(1./RW-1./RE)/H**2
      DO 50 NV=NS,NF
      JJ=0
      J=NV+1
      DO 60 I=J,ND
      YI(I)=(PE(I)**2-PW(I)**2+2.*B(NV)*(PE(I)-PW(I)))/
     1(QQ(I)*MUTZ(I))
      XI(I)=QQ(I)/MU(I)
```

DISCRIMINATION MAIN ...(CONT'D)

```
      JJ=JJ+1
      YIM(JJ)=YI(I)
      XIM(JJ)=XI(I)
   60 CONTINUE
      NI=JJ
      CALL LFIT(NI,YIM,XIM,A1,A0,S2,R)
      KI(NI)=CA/A0
      FB(NI)=A1/CB
      S2I(NI)=S2
      RI(NI)=R
      ERRK(NV)=100.*(KV(NV)-KI(NI))/KV(NV)
      SLEFF=2.*B(NV)*(PE(NV)-PW(NV))
      VSEFF=CA*MUTZ(NV)*QO(NV)/KV(NV)
      INEFF=CB*FB(NI)*MUTZ(NV)*QO(NV)**2/MU(NV)
      NRE(NV)=INEFF/(VSEFF-SLEFF)
      IF(NI-4)66,50,50
   50 WRITE(6,70)NV,B(NV),KV(NV),KI(NI),FB(NI),NRE(NV)
   70 FORMAT(8X,I4,1X,5(E11.4,1X))

C     .... DATA DISCRIMINATION METHODS ....                    *
C     ICONT=1,METHOD 1 . FINDING THE BEST FIT,I.E. RV(MAX)*
C     ICONT=2,METHOD 2 . MATCHING K VALUES,I.E. KV AND KI *
C     ICONT=3,METHOD 3 . REYNOLDS NUMBER CRITERION         *

   66 DO 22 ICONT=1,3
      CALL METOD(NS,NF,ND,ICONT,RV,ERRK,NRE,NN,NI)
      GO TO(80,85,88),ICONT
   80 WRITE(6,90)NN,RV(NN),NI,RI(NI)
   90 FORMAT(/10X,'DATA DISCRIMINATION METHODS'//10X,
     1'METHOD 1. FINDING THE BEST FIT FOR THE KLINKENBERG'
     2' PLOT'/20X,'I.E. THE MAXIMUM CORRELATION COEFFICIE'
     3'NT'/30X,'NV=',I3,5X,'R(NV)=',F6.3,1X,'(MAXIMUM)'
     4/30X,'NI=',I3,5X,'R(NI)=',F6.3)
      GO TO 22
   85 WRITE(6,95)NN,NI,ERRK(NN)
   95 FORMAT(/10X,'METHOD 2. MATCHING THE VALUES OF K '
     1'FROM THE KLINKENBERG'/26X,'AND MODIFIED VISCO-'
     3'INERTIAL PLOTS'/20X,'I.E. THE MINIMUM RELATIVE '
     4'ERROR BETWEEN KV AND KI'/30X,'NV=',I3/30X,'NI=',I3/
     524X,'ERRK(NV)=',E11.4,1X,'(MIN. PERCENT)')
      GO TO 22
   88 WRITE(6,99)NN,NI,NRE(NN)
   99 FORMAT(/10X,'METHOD 3. THE REYNOLDS NUMBER CRITERION'
     1',I.E. THE POINTS ARE'/20X,'IN VISCOUS-REGION IF'
     2' NRE IS LESS THAN 0.01'/30X,'NV=',I3/30X,'NI=',I3/
     325X,'NRE(NV)=',E11.4)
   22 CONTINUE
    1 CONTINUE
      CALL EXIT
      END
```

```
      SUBROUTINE  LFIT

C     ************************************************************
C                                                                  *
C     IT PERFORMS A LINEAR LEAST SQUARES ESTIMATION FOR            *
C     THE GIVEN DATA PAIRS OF Y AND X,(Y,X)                        *
C                                                                  *
C     IT ALSO CALCULATES THE COEFFICIENT OF VARIATION, R           *
C     AND THE VARIANCE, S2 ,OF THE FIT                             *
C                                                                  *
C     ************************************************************
C
      SUBROUTINE LFIT(N,Y,X,A1,A0,S2,R)

      DIMENSION X(40),Y(40)
      XSUM=0.
      DO 5 I=1,N
    5 XSUM=XSUM+X(I)
      RN=N
      XBAR=XSUM/RN
      SUMZ=0.
      SUMX=0.
      SUMY=0.
      SUM2X=0.
      SUM2Y=0.
      SUMXY=0.
      DO 10 I=1,N
      SUMZ=SUMZ+X(I)-XBAR
      SUMX=SUMX+X(I)
      SUMY=SUMY+Y(I)
      SUM2X=SUM2X+X(I)**2
      SUM2Y=SUM2Y+Y(I)**2
   10 SUMXY=SUMXY+X(I)*Y(I)
      C=N
      A1=(C*SUMXY-SUMX*SUMY)/(C*SUM2X-SUMX**2)
      A0=(SUMY-A1*SUMX)/C
      S2=(SUM2Y-A1*SUMXY-A0*SUMY)/(C-2.)
      S0=S2*(1./C+XBAR**2/SUMZ)
      S1=S2/SUMZ
      R=(C*SUMXY-SUMX*SUMY)/SQRT((C*SUM2X-SUMX**2)
     1*(C*SUM2Y-SUMY**2))
      RETURN
      END
```

SUBROUTINE METOD

```
C     ************************************************************
C                                                                  *
C     IT DISCRIMINATES THE FLOW DATA BY APPLYING THE               *
C        VARIOUS DATA SPLITTIG METHODS AS SUGGESTED                *
C                                                                  *
C     ************************************************************

      SUBROUTINE METOD(NS,NF,ND,ICONT,RV,ERRK,NRE,NN,NI)
      REAL NRE(40)
      DIMENSION RV(40),ERRK(40)
      GO TO(11,22,33),ICONT

C     ..... METHOD 1 .....

   11 RMAX=RV(NS)
      LL=NS+1
      DO 10 NV=LL,NF
      IF(RMAX-RV(NV))15,15,10
   15 RMAX=RV(NV)
      NN=NV
      NI=ND-NN
   10 CONTINUE
      GO TO 20

C     ..... METHOD 2 .....

   22 SMALL=ERRK(NS)
      LL=NS+1
      DO 25 NV=LL,NF
      IF(ABS(SMALL)-ABS(ERRK(NV)))25,25,30
   30 SMALL=ERRK(NV)
      NN=NV
      NI=ND-NN
   25 CONTINUE
      GO TO 20

C     ..... METHOD 3 .....

   33 DO 40 NV=NS,NF
      IF(NRE(NV)-0.01)40,44,45
   44 NN=NV
      NI=ND-NN
      GO TO 20
   45 NN=NV-1
      NI=ND-NN
      GO TO 20
   40 CONTINUE
   20 RETURN
      END
```

APPENDIX C

TABULATED RAW EXPERIMENTAL DATA

APPENDIX C

Remarks:

The following Tables C-1 through C-26 contain the observed flow data for the rock samples used under different confinement pressures.

Each table has an identification code which indicates the applied net confinement pressure P_{nc} (psig), the run number on that particular sample and the gas used as the flowing fluid. This code is

P_{nc} - Run No., Test Fluid

All the runs are conducted under the Test Method Case I unless specified otherwise.

These data are tabulated under a total of eight columns, with abbreviated descriptions, which may be explained in detail as follows:

Column Number	Explanation
1	Serial number indicating the observed number of data points, ND.
2	Barometric Pressure, P_b, in centimeters of Hg.
3	Upstream Pressure, P_e, in psig or inches of Hg*.
4	Downstream Pressure, P_w, in inches of H_2O or Hg* or in psig**.
5	Upstream Thermocouple Reading, TC_e, in millivolts.
6	Downstream Thermocouple Reading, TC_w, in millivolts.

Column Number	Explanation
7	Volume of gas, V , passed during the time recorded in column (8), in cc and in cuft* or cuft** (correction due to calibration).
8	Arithmetic mean time, t_m, for several repeated measurements, in seconds.

TABLE C-1

OBSERVED DATA FOR SAMPLE LS-1

600-1A, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.25	1.90*	0.00	0.841	0.831	100.0	610.0
2	70.25	2.50*	0.00	0.850	0.835	100.0	460.0
3	70.25	3.50*	0.00	0.850	0.835	100.0	325.0
4	70.25	4.50*	0.00	0.860	0.845	100.0	248.4
5	70.25	5.70*	0.00	0.860	0.845	100.0	194.0
6	70.25	7.10*	0.10	0.860	0.845	100.0	154.0
7	70.25	8.70*	0.15	0.865	0.850	100.0	122.4
8	70.15	11.00*	0.20	0.870	0.855	100.0	93.8
9	70.15	13.00*	0.30	0.868	0.854	100.0	78.0
10	70.15	15.50*	0.35	0.868	0.854	100.0	63.6
11	70.15	19.00*	0.50	0.868	0.854	100.0	49.5
12	70.15	24.075*	0.65	0.868	0.854	100.0	37.4
13	70.15	30.00*	0.90	0.868	0.854	100.0	28.2
14	70.15	37.00*	1.10	0.870	0.855	100.0	21.5
15	70.00	45.05*	1.70	0.880	0.870	100.0	16.4
16	70.00	54.20*	5.90	0.890	0.880	1.040**	3780.0
17	70.00	32.05	0.50*	0.890	0.880	1.031**	2899.8
18	70.00	36.00	0.55*	0.900	0.880	1.032**	2460.0
19	70.00	40.00	0.625*	0.900	0.880	1.027**	2115.0
20	70.00	45.00	0.70*	0.900	0.880	1.022**	1778.4
21	70.00	51.975	0.85*	0.900	0.880	1.021**	1444.8
22	70.00	60.00	1.05*	0.900	0.850	1.021**	1172.4
23	70.00	70.00	1.30*	0.900	0.850	1.013**	927.6
24	70.00	81.975	1.85*	0.900	0.850	1.012**	733.8
25	70.00	94.00	2.45*	0.900	0.850	1.010**	600.0
26	70.00	120.00	4.25*	0.900	0.865	1.000*	414.0
27	70.00	150.00	7.30*	0.900	0.865	1.000*	297.0
28	70.00	190.00	13.05*	0.900	0.865	1.000*	211.5
29	70.00	239.80	22.15*	0.910	0.875	1.000*	115.5
30	70.00	300.40	35.70*	0.910	0.875	1.000*	110.7
31	70.00	399.40	30.60**	0.910	0.880	1.000*	77.0

TABLE C-2

OBSERVED DATA FOR SAMPLE LS-1

600-1B, Ar

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	69.85	2.30*	0.00	0.825	0.810	100.0	628.0
2	69.85	3.00*	0.00	0.830	0.815	100.0	481.0
3	69.85	4.10*	0.00	0.830	0.815	100.0	342.0
4	69.85	5.30*	0.00	0.830	0.815	100.0	262.0
5	69.85	6.75*	0.00	0.830	0.820	100.0	202.0
6	69.85	8.30*	0.00	0.830	0.820	100.0	161.0
7	69.85	10.05*	0.00	0.830	0.820	100.0	130.0
8	69.80	12.55*	0.10	0.830	0.820	100.0	101.0
9	69.80	14.95*	0.25	0.830	0.820	100.0	82.5
10	69.80	18.07*	0.35	0.830	0.820	100.0	66.0
11	69.90	20.15*	0.50	0.780	0.772	100.0	57.2
12	69.90	25.25*	0.60	0.786	0.776	100.0	43.6
13	69.90	30.10*	0.80	0.792	0.782	100.0	35.0
14	69.90	38.15*	1.15	0.795	0.785	100.0	25.8
15	69.90	48.00*	2.80	0.800	0.790	100.0	19.0
16	69.90	58.95*	5.00	0.800	0.790	1.030**	4200.0
17	69.90	31.95	0.40*	0.810	0.798	1.018**	3624.0
18	69.90	36.05	0.49*	0.812	0.800	1.009**	3030.0
19	69.60	40.20	0.38*	0.780	0.765	1.009**	2583.0
20	69.60	45.00	0.48*	0.782	0.772	1.000*	2166.0
21	69.60	52.00	0.60*	0.790	0.750	1.000*	1776.0
22	69.60	60.00	0.80*	0.792	0.765	1.000*	1431.0
23	69.60	70.00	1.10*	0.792	0.765	1.000*	1138.5
24	69.60	82.00	1.52*	0.792	0.772	1.000*	897.0
25	69.60	94.00	2.20*	0.800	0.780	1.000*	730.8
26	69.60	120.0	4.10*	0.800	0.780	1.000*	504.0
27	69.60	150.00	7.15*	0.800	0.780	1.000*	364.2
28	69.60	190.00	12.65*	0.800	0.780	1.000*	256.8
29	69.60	240.00	21.55*	0.810	0.790	1.000*	182.4
30	69.60	300.20	34.75*	0.810	0.790	1.000*	133.9

TABLE C-3

OBSERVED DATA FOR SAMPLE LS-1

600-1C, CH_4

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	69.75	1.50*	0.00	0.860	0.850	100.0	500.0
2	69.75	2.30*	0.00	0.860	0.850	100.0	328.0
3	69.75	3.125*	0.00	0.860	0.850	100.0	234.0
4	69.75	4.050*	0.00	0.860	0.850	100.0	178.5
5	69.75	5.50*	0.00	0.860	0.850	100.0	130.0
6	69.75	6.75*	0.00	0.860	0.850	100.0	104.0
7	69.75	18.30*	0.00	0.860	0.850	100.0	83.0
8	69.75	10.10*	0.00	0.860	0.850	100.0	66.0
9	69.75	12.55*	0.10	0.860	0.850	100.0	51.4
10	69.75	14.90*	0.15	0.860	0.850	100.0	42.5
11	69.75	18.10*	0.30	0.860	0.850	100.0	33.6
12	69.75	21.10*	0.40	0.860	0.850	100.0	28.0
13	69.75	25.85*	0.60	0.860	0.850	100.0	21.8
14	69.80	15.00	0.175*	0.855	0.845	1.015**	5109.0
15	69.80	18.00	0.240*	0.865	0.845	1.023**	4086.6
16	69.80	21.00	0.275*	0.860	0.850	1.019**	3330.0
17	69.80	25.00	0.30*	0.860	0.850	1.013**	2625.0
18	69.80	30.20	0.40*	0.855	0.845	1.010**	2025.0
19	69.80	35.00	0.48*	0.855	0.845	1.001**	1635.0
20	69.80	40.00	0.52*	0.860	0.850	1.003**	1353.0
21	69.80	45.00	0.69*	0.860	0.850	1.000*	1150.2
22	69.80	52.00	0.82*	0.860	0.850	1.000*	927.6
23	69.80	60.00	1.06*	0.860	0.850	1.000*	757.2
24	69.80	70.00	1.425*	0.860	0.850	1.000*	603.0
25	69.80	82.00	2.05*	0.865	0.850	1.000*	477.0
26	69.80	94.15	2.85*	0.865	0.850	1.000*	390.0
27	69.80	120.00	5.225*	0.870	0.850	1.000*	274.2
28	69.80	150.00	8.95*	0.870	0.850	1.000*	199.2
29	69.80	190.00	15.625*	0.865	0.850	1.000*	141.0
30	69.80	240.00	26.25*	0.865	0.850	1.000*	103.5
31	69.80	300.20	41.65*	0.865	0.850	1.000*	77.52

TABLE C-4

OBSERVED DATA FOR SAMPLE LS-1

800-2, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	69.60	1.00*	0.00	0.895	0.881	100.0	1187.00
2	69.60	1.40*	0.00	0.900	0.890	100.0	844.00
3	69.60	2.00*	0.00	0.900	0.890	100.0	586.50
4	69.60	3.00*	0.00	0.895	0.850	100.0	384.60
5	69.60	4.00*	0.00	0.880	0.852	100.0	284.20
6	69.65	5.00*	0.00	0.900	0.890	100.0	223.80
7	69.65	7.00*	0.00	0.920	0.894	100.0	155.75
8	69.65	9.00*	0.15	0.920	0.894	100.0	117.60
9	69.65	12.00*	0.25	0.920	0.894	100.0	85.50
10	69.65	15.00*	0.30	0.920	0.894	100.0	65.50
11	69.65	19.00*	0.40	0.906	0.894	100.0	49.50
12	69.65	25.00*	0.65	0.906	0.894	100.0	35.495
13	69.80	30.00*	0.80	0.800	0.785	100.0	28.011
14	69.80	36.00*	1.10	0.800	0.785	100.0	22.00
15	69.80	44.00*	1.40	0.800	0.785	100.0	16.876
16	69.80	55.90*	6.10	0.800	0.785	1.028**	3591.00
17	69.75	32.10	0.39*	0.802	0.788	1.018**	2859.00
18	69.75	40.05	0.50*	0.805	0.790	1.009**	2079.00
19	69.75	50.00	0.675*	0.805	0.790	1.007**	1506.00
20	69.75	60.00	0.935*	0.805	0.790	1.000*	1149.00
21	69.75	70.025	1.250*	0.805	0.790	1.000*	912.00
22	69.75	80.00	1.610*	0.810	0.790	1.000*	750.00
23	69.75	95.05	2.40*	0.810	0.790	1.000*	581.82
24	69.75	105.20	3.05*	0.810	0.790	1.000*	499.50
25	69.75	120.20	4.30*	0.810	0.790	1.000*	408.90
26	69.75	135.00	5.70*	0.810	0.790	1.000*	345.00
27	69.75	150.00	7.45*	0.810	0.790	1.000*	294.90
28	69.75	175.00	10.80*	0.810	0.790	1.000*	235.80
29	69.75	200.10	14.58*	0.810	0.790	1.000*	194.04
30	69.40	250.15	24.25*	0.900	0.875	1.000*	142.02
31	69.75	304.60	36.80*	0.810	0.790	1.000*	108.00

TABLE C-5

OBSERVED DATA FOR SAMPLE LS-1

1000-3, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	69.65	1.00*	0.00	0.890	0.880	100.0	1181.00
2	69.65	1.50*	0.00	0.890	0.880	100.0	767.00
3	69.65	2.00*	0.00	0.890	0.880	100.0	576.50
4	69.65	3.00*	0.00	0.896	0.880	100.0	379.20
5	69.65	4.00*	0.00	0.896	0.880	100.0	281.50
6	69.625	5.00*	0.10	0.845	0.840	100.0	221.50
7	69.625	7.00*	0.15	0.845	0.840	100.0	115.40
8	69.625	9.00*	0.20	0.845	0.840	100.0	117.80
9	69.625	12.00*	0.25	0.860	0.850	100.0	84.70
10	69.625	15.00*	0.35	0.860	0.850	100.0	66.00
11	69.70	20.00*	0.45	0.860	0.850	100.0	46.80
12	69.70	25.00*	0.60	0.865	0.855	100.0	35.20
13	69.70	32.00*	0.90	0.865	0.855	100.0	25.70
14	69.70	40.00*	1.20	0.865	0.855	100.00	19.25
15	69.70	50.00*	1.80	0.865	0.855	100.0	14.30
16	69.70	30.20	5.40	0.870	0.855	1.027**	3146.04
17	69.70	35.00	6.30	0.870	0.855	1.022**	2105.40
18	69.70	40.00	7.30	0.870	0.855	1.017**	2548.35
19	69.90	50.05	10.00	0.875	0.865	1.016**	1530.00
20	69.90	60.00	13.40	0.875	0.865	1.000*	1164.00
21	69.90	72.00	18.60	0.875	0.865	1.000*	889.02
22	69.90	82.00	24.20	0.875	0.865	1.000*	733.02
23	69.90	95.00	2.40*	0.885	0.835	1.000*	590.04
24	69.90	110.15	3.45*	0.885	0.855	1.000*	472.02
25	69.90	126.00	4.85*	0.885	0.855	1.000*	386.40
26	69.90	140.00	6.25*	0.885	0.855	1.000*	331.50
27	69.90	165.00	9.35*	0.885	0.855	1.000*	259.02
28	69.90	200.00	14.10*	0.895	0.870	1.000*	196.38
29	69.90	251.30	24.40*	0.895	0.870	1.000*	142.74
30	69.90	300.20	35.20*	0.895	0.870	1.000*	111.80

TABLE C-6

OBSERVED DATA FOR SAMPLE LS-1

1200-4, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	69.30	1.0*	0.00	0.890	0.875	100.0	1194.00
2	69.30	2.0*	0.00	0.890	0.875	100.0	585.00
3	69.30	3.0*	0.00	0.890	0.875	100.0	373.50
4	69.35	4.0*	0.10	0.885	0.870	100.0	278.40
5	69.35	6.0*	0.15	0.885	0.870	100.0	180.80
6	69.35	8.0*	0.20	0.885	0.870	100.0	133.40
7	69.35	11.0*	0.25	0.885	0.870	100.0	93.50
8	69.35	14.0*	0.35	0.885	0.870	100.0	70.80
9	69.35	18.0*	0.50	0.885	0.870	100.0	52.20
10	69.35	24.0*	0.625	0.885	0.870	100.0	36.80
11	69.45	30.0*	0.80	0.885	0.870	100.0	28.10
12	69.45	40.0*	1.30	0.885	0.870	100.0	19.235
13	69.45	52.0*	1.95	0.885	0.870	100.0	13.40
14	70.10	35.20	5.50	0.875	0.865	1.024**	2523.00
15	70.10	45.00	7.60	0.875	0.865	1.018**	1770.00
16	70.60	56.80	0.80*	0.880	0.870	1.003**	1263.40
17	70.60	70.00	1.20*	0.880	0.870	1.000*	928.02
18	70.60	85.00	1.81*	0.880	0.870	1.000*	696.00
19	70.60	100.00	2.63*	0.880	0.870	1.000*	548.40
20	70.60	120.00	4.22*	0.885	0.870	1.000*	417.48
21	70.60	140.00	6.22*	0.885	0.870	1.000*	332.04
22	70.60	170.10	9.975*	0.885	0.870	1.000*	250.02
23	70.60	200.10	14.65*	0.885	0.870	1.000*	198.50
24	70.60	249.80	24.50*	0.895	0.870	1.000*	144.87
25	70.60	300.20	35.45*	0.895	0.870	1.000*	111.67

TABLE C-7

OBSERVED DATA FOR SAMPLE LS-1

800-5, N_2

at P_e = 300 psig (Test Method: Case II)

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.00	299.50	17.25**	0.780	0.760	1.00*	111.78
2	70.00	300.00	40.15**	0.780	0.760	1.00*	114.00
3	70.00	300.00	60.00**	0.780	0.760	1.00*	116.16
4	70.00	300.00	80.00**	0.780	0.760	1.00*	119.46
5	70.00	301.00	110.20**	0.780	0.760	1.00*	125.85
6	70.00	301.00	139.80**	0.780	0.760	1.00*	136.20
7	70.00	301.50	170.00**	0.780	0.760	1.00*	151.50
8	70.00	301.50	200.80**	0.780	0.760	1.00*	177.00
9	70.00	301.50	219.65**	0.785	0.770	1.00*	202.80
10	70.00	302.00	240.00**	0.785	0.770	1.00*	250.80
11	70.00	300.00	250.00**	0.765	0.750	1.00*	292.98
12	70.00	300.00	259.50**	0.765	0.750	1.00*	343.98
13	70.00	300.00	264.60**	0.765	0.750	1.00*	382.20
14	70.00	300.00	269.60**	0.765	0.750	1.00*	432.00
15	70.00	300.00	276.00**	0.775	0.760	1.00*	526.50
16	70.00	300.00	280.00**	0.775	0.760	1.00*	616.50
17	70.00	300.00	284.00**	0.775	0.760	1.00*	754.50
18	70.00	300.00	288.00**	0.775	0.760	1.00*	963.00
19	70.00	300.00	290.00**	0.775	0.760	1.00*	1140.00
20	70.00	300.50	292.00**	0.805	0.790	1.00*	1434.00
21	70.00	300.50	294.00**	0.790	0.780	1.00*	1887.00
22	70.00	300.50	296.00**	0.790	0.780	1.00*	2871.00
23	70.00	300.50	297.20**	0.790	0.780	1.00*	4140.00
24	70.00	300.50	298.00**	0.790	0.780	100.0	20.00
25	70.00	300.50	298.50**	0.790	0.780	100.0	34.50
26	70.00	300.50	299.00**	0.790	0.780	100.0	67.00
27	70.00	300.00	299.20**	0.790	0.780	100.0	122.50
28	70.00	301.00	300.00**	0.790	0.780	100.0	265.00
29	70.00	300.00	299.40**	0.790	0.780	100.0	619.00

TABLE C-8

OBSERVED DATA FOR SAMPLE LS-1

1000-6 N_2

at P_e = 300 psig (Test Method: Case II)

1 (ND)	2 (P_b)	3 (P_e)	3 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	69.80	299.50	17.15**	0.900	0.870	1.00*	112.33
2	69.80	300.00	92.35**	0.900	0.870	1.00*	120.88
3	69.80	301.75	150.30**	0.900	0.870	1.00*	140.04
4	69.80	300.00	180.00**	0.900	0.880	1.00*	160.02
5	69.80	300.00	209.60**	0.900	0.880	1.00*	190.72
6	69.80	300.00	230.00**	0.900	0.880	1.00*	226.68
7	69.80	300.00	250.00**	0.900	0.880	1.00*	288.80
8	69.80	300.00	260.00**	0.900	0.880	1.00*	342.77
9	69.80	300.50	270.80**	0.900	0.880	1.00*	438.00
10	69.80	300.50	280.20**	0.900	0.880	1.00*	602.04
11	69.80	300.50	286.00**	0.900	0.880	1.00*	802.08
12	69.30	300.50	290.00**	0.900	0.885	1.00*	1092.60
13	69.30	300.50	293.90**	0.900	0.885	1.00*	1727.58
14	69.30	300.50	296.20**	0.900	0.885	1.00*	2652.96
15	69.30	300.50	297.00**	0.900	0.885	1.00*	3430.02
16	69.30	300.50	298.20**	0.900	0.885	1.00*	5400.00
17	69.25	300.50	298.60**	0.900	0.890	100.0	25.40
18	69.25	300.50	299.00**	0.900	0.890	100.0	35.00
19	69.25	300.50	299.60**	0.900	0.890	100.0	106.20
20	69.25	300.50	299.80**	0.900	0.890	100.0	353.00

TABLE C-9

OBSERVED DATA FOR SAMPLE LS-2

400-1, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.55	5.0	0.00	0.780	0.765	100.00	879.63
2	70.55	8.0	0.00	0.785	0.770	100.00	512.57
3	70.40	10.0	0.00	0.845	0.845	100.00	384.60
4	70.40	16.0	0.00	0.854	0.852	100.00	218.34
5	70.40	20.0	0.00	0.834	0.831	100.00	162.82
6	70.40	22.0	0.00	0.840	0.840	100.00	142.66
7	70.40	28.0	0.00	0.835	0.830	100.00	101.97
8	70.40	40.0	0.00	0.835	0.840	100.00	59.70
9	70.20	50.0	0.35	0.770	0.750	100.00	42.10
10	70.40	60.0	0.00	0.830	0.830	100.00	31.62
11	70.40	75.0	0.75	0.845	0.830	1.00*	4352.77
12	70.60	100.0	0.03*	0.812	0.812	1.00*	3839.17
13	70.60	120.0	0.05*	0.817	0.817	1.00*	2806.90
14	70.60	140.0	0.05*	0.817	0.817	1.00*	2146.10
15	70.60	160.0	0.075	0.825	0.820	1.00*	1700.07
16	70.60	180.0	0.10*	0.825	0.820	1.00*	1386.57
17	70.60	200.0	0.10*	0.825	0.820	1.00*	1155.92
18	70.40	230.0	0.60*	0.810	0.790	1.00*	895.24
19	70.55	300.0	1.00*	0.810	0.790	1.00*	557.11
20	70.55	450.0	2.80*	0.810	0.790	1.00*	266.97

TABLE C-10

OBSERVED DATA FOR SAMPLE LS-2

400-2, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.00	4.00*	0.000	0.980	0.970	100.00	1800.68
2	70.00	6.00*	0.000	0.980	0.970	100.00	1171.14
3	70.00	8.00*	0.000	0.980	0.970	100.00	865.21
4	69.90	10.00*	0.000	0.980	0.970	100.00	674.72
5	69.90	12.00*	0.000	0.980	0.970	100.00	551.18
6	69.90	15.00*	0.000	0.970	0.970	100.00	428.00
7	69.90	18.00*	0.000	0.970	0.970	100.00	347.00
8	69.90	22.00*	0.000	0.970	0.970	100.00	274.50
9	69.90	26.00*	0.100	0.970	0.970	100.00	224.00
10	69.90	30.00*	0.150	0.9725	0.9675	100.00	187.70
11	69.90	35.00*	0.175	0.9725	0.9675	100.00	155.00
12	69.90	42.00*	0.200	0.9725	0.9675	100.00	121.90
13	69.90	50.00*	0.250	0.9725	0.9675	100.00	96.90
14	69.90	29.00	0.300	0.9725	0.9675	100.00	77.04
15	69.90	35.00	0.400	0.9725	0.9675	100.00	59.20
16	69.80	40.80	0.450	0.970	0.960	100.00	47.33
17	69.80	48.80	0.600	0.970	0.960	100.00	36.30
18	69.80	58.80	0.850	0.970	0.960	100.00	27.22
19	69.80	73.80	1.250	0.970	0.960	100.00	18.82
20	69.80	88.80	1.800	0.970	0.960	100.00	13.86
21	69.80	110.00	2.900	0.970	0.960	100.00	9.67
22	69.80	130.00	7.200	0.970	0.960	2.50*	5104.14
23	69.80	150.00	9.100	0.970	0.960	2.50*	3990.37
24	69.80	170.00	11.500	0.970	0.960	2.50*	3189.86
25	69.80	190.00	14.400	0.970	0.960	2.50*	2610.92
26	69.80	210.00	18.200	0.970	0.960	2.50*	2250.00
27	69.80	240.00	1.850*	0.970	0.960	2.50*	1711.07
28	69.80	280.00	2.850*	0.970	0.960	2.50*	1285.59
29	69.80	320.00	4.350*	0.970	0.960	2.50*	1005.00
30	69.80	360.00	6.300*	0.970	0.960	2.50*	807.00
31	69.80	400.00	8.700*	0.970	0.960	2.50*	662.00
32	69.80	460.00	13.100*	0.970	0.960	2.50*	511.00
33	69.80	549.00	21.800*	0.970	0.960	2.50*	371.50
24	69.80	646.00	34.250*	0.970	0.960	2.50*	273.00
35	69.80	783.00	56.300*	0.970	0.960	2.50*	191.00

TABLE C-11

OBSERVED DATA FOR SAMPLE LS-2

600-3, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.10	4.00*	0.000	0.870	0.860	100.00	2286.07
2	70.10	6.00*	0.000	0.870	0.860	100.00	1493.22
3	70.10	8.00*	0.000	0.870	0.860	100.00	1102.44
4	70.10	10.00*	0.000	0.870	0.860	100.00	865.03
5	70.13	12.00*	0.000	0.870	0.860	100.00	705.96
6	70.13	15.00*	0.000	0.870	0.860	100.00	549.47
7	70.13	18.00*	0.000	0.870	0.860	100.00	445.87
8	70.13	22.00*	0.000	0.870	0.860	100.00	353.15
9	70.13	26.00*	0.000	0.870	0.860	100.00	289.50
10	70.13	30.00*	0.050	0.870	0.860	100.00	242.27
11	70.13	34.95*	0.100	0.870	0.860	100.00	199.50
12	70.13	39.98*	0.150	0.870	0.860	100.00	168.00
13	70.13	48.00*	0.175	0.870	0.860	100.00	132.06
14	70.13	58.00*	0.200	0.870	0.860	100.00	102.50
15	70.13	32.00	0.250	0.873	0.862	100.00	87.05
16	70.13	38.00	0.300	0.873	0.862	100.00	67.91
17	70.13	46.00	0.400	0.873	0.862	100.00	51.28
18	70.13	56.00	0.550	0.873	0.862	100.00	37.89
19	70.13	66.00	0.750	0.873	0.862	100.00	29.34
20	70.15	76.00	1.000	0.873	0.862	100.00	23.30
21	70.15	90.00	1.550	0.873	0.862	100.00	17.60
22	70.15	110.00	2.000	0.873	0.862	100.00	12.62
23	70.15	130.20	8.200	0.873	0.862	1.00*	2656.91
24	70.15	150.00	9.200	0.873	0.862	1.00*	2081.06
25	70.15	170.00	10.950	0.878	0.867	1.00*	1659.06
26	70.15	190.00	13.000	0.878	0.867	1.00*	1358.06
27	70.15	209.90	15.400	0.878	0.867	1.00*	1141.75
28	70.15	239.80	1.475*	0.878	0.867	1.00*	892.98
29	70.15	280.00	2.075*	0.878	0.867	1.00*	666.91
30	70.20	320.10	2.950*	0.880	0.830	1.00*	521.00
31	70.20	360.00	4.150*	0.880	0.830	1.00*	418.00
32	70.20	400.00	5.650*	0.880	0.830	1.00*	342.50
33	70.20	460.00	8.650*	0.880	0.830	1.00*	264.00
34	70.20	548.00	14.700*	0.880	0.830	1.00*	190.00
35	70.20	650.00	24.400*	0.900	0.870	1.00*	138.00

TABLE C-12

OBSERVED DATA FOR SAMPLE LS-2

800-4, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.55	8.00*	0.000	0.840	0.835	100.00	1177.75
2	70.55	11.00*	0.000	0.840	0.835	100.00	832.96
3	70.55	15.00*	0.000	0.850	0.845	100.00	590.00
4	70.55	20.00*	0.000	0.850	0.845	100.00	425.00
5	70.55	25.00*	0.000	0.850	0.845	100.00	327.00
6	70.55	30.00*	0.000	0.850	0.845	100.0	261.50
7	70.55	37.98*	0.050	0.850	0.835	100.00	193.86
8	70.55	48.00*	0.100	0.850	0.835	100.00	143.55
9	70.55	57.98*	0.175	0.850	0.835	100.00	111.50
10	70.55	34.00	0.225	0.850	0.835	100.00	86.37
11	70.55	40.00	0.300	0.850	0.835	100.00	68.40
12	70.50	48.00	0.400	0.860	0.850	100.00	52.27
13	70.50	60.00	0.600	0.860	0.850	100.00	37.50
14	70.50	74.00	0.900	0.860	0.850	100.00	26.80
15	70.50	90.00	1.250	0.860	0.850	100.00	19.50
16	70.50	110.00	1.900	0.860	0.850	100.00	14.03
17	70.50	130.15	7.300	0.860	0.850	1.00*	2980.50
18	70.50	150.00	8.400	0.860	0.850	1.00*	2332.80
19	70.50	180.00	10.600	0.860	0.850	1.00*	1692.78
20	70.50	210.00	13.400	0.860	0.850	1.00*	1279.60
21	70.50	239.90	17.200	0.870	0.855	1.00*	1006.20
22	70.50	279.70	1.700*	0.870	0.855	1.00*	756.50
23	70.50	320.10	2.450*	0.870	0.855	1.00*	587.00
24	70.50	360.00	3.400*	0.870	0.855	1.00*	471.50
25	70.45	420.00	5.350*	0.850	0.830	1.00*	355.00
26	70.45	509.00	9.800*	0.850	0.830	1.00*	245.00
27	70.45	644.00	20.200*	0.850	0.830	1.00*	156.25
28	70.45	827.00	41.500*	0.850	0.830	1.00*	98.00
29	70.45	1006.00	35.200**	0.850	0.830	1.00*	67.40

TABLE C-13

OBSERVED DATA FOR SAMPLE LS-2

1000-5, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.45	10.00*	0.000	0.865	0.855	100.00	943.92
2	70.45	12.00*	0.000	0.865	0.855	100.00	772.82
3	70.45	15.00*	0.000	0.870	0.865	100.00	603.80
4	70.45	20.00*	0.000	0.870	0.865	100.00	435.65
5	70.45	25.00*	0.000	0.870	0.865	100.00	334.85
6	70.45	30.00*	0.000	0.870	0.865	100.00	268.25
7	70.45	36.00*	0.050	0.870	0.865	100.00	214.50
8	70.45	44.00*	0.100	0.870	0.865	100.00	166.99
9	70.45	56.00*	0.175	0.870	0.865	100.00	121.05
10	70.45	34.00	0.250	0.885	0.875	100.00	90.69
11	70.45	40.00	0.300	0.885	0.875	100.00	71.91
12	70.45	50.00	0.400	0.885	0.875	100.00	51.81
13	70.45	60.00	0.600	0.885	0.875	100.00	38.90
14	70.40	74.00	0.800	0.905	0.890	100.00	28.35
15	70.40	90.00	1.200	0.905	0.890	100.00	20.76
16	70.40	109.80	1.800	0.905	0.890	100.00	14.95
17	70.18	130.05	2.400	0.860	0.835	100.00	11.20
18	70.18	150.00	6.650	0.860	0.835	1.00*	2477.64
19	70.18	179.90	8.400	0.875	0.850	1.00*	1793.01
20	70.18	210.00	10.950	0.875	0.850	1.00*	1380.98
21	70.25	240.00	14.200	0.890	0.880	1.00*	1084.13
22	70.25	279.80	19.800	0.890	0.880	1.00*	825.86
23	70.25	320.00	2.050*	0.890	0.880	1.00*	642.25
24	70.25	360.00	2.800*	0.890	0.880	1.00*	511.17
25	70.25	420.00	4.500*	0.910	0.900	1.00*	388.09
26	70.25	495.00	7.600*	0.910	0.900	1.00*	283.96
27	70.25	624.50	15.750*	0.910	0.900	1.00*	181.50
28	70.25	775.00	30.200*	0.910	0.900	1.00*	120.00
29	70.25	876.50	42.150*	0.910	0.900	1.00*	95.00

TABLE C-14

OBSERVED DATA FOR SAMPLE LS-3-A

400-1, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.10	2.00*	0.150	0.770	0.770	100.00	611.47
2	70.10	3.00*	0.175	0.770	0.770	100.00	400.95
3	70.10	4.00*	0.175	0.770	0.770	100.00	295.98
4	70.15	5.00*	0.200	0.780	0.780	100.00	233.45
5	70.15	6.00*	0.200	0.780	0.780	100.00	191.63
6	70.15	8.00*	0.225	0.780	0.780	100.00	139.44
7	70.15	11.00*	0.300	0.780	0.780	100.00	97.20
8	70.15	15.00*	0.375	0.785	0.780	100.00	67.63
9	70.15	20.00*	0.500	0.785	0.780	100.00	47.70
10	70.15	26.00*	0.700	0.785	0.780	100.00	34.26
11	70.20	34.00*	1.000	0.795	0.785	100.00	23.99
12	70.20	44.00*	1.450	0.795	0.785	100.00	16.82
13	70.20	56.00*	1.200	0.795	0.785	100.00	11.88
14	70.27	34.10	5.300	0.800	0.800	1.00*	2444.92
15	70.27	40.10	6.700	0.800	0.800	1.00*	1903.34
16	70.30	48.00	9.200	0.810	0.800	1.00*	1431.52
17	70.30	58.00	11.150	0.810	0.800	1.00*	1053.52
18	70.30	70.00	20.100	0.810	0.800	1.00*	772.53
19	70.37	84.00	2.400*	0.815	0.805	1.00*	570.00
20	70.37	99.80	3.785*	0.815	0.805	1.00*	427.00
21	70.37	120.10	6.350*	0.815	0.805	1.00*	312.00
22	70.40	140.00	9.750*	0.820	0.805	1.00*	341.01
23	70.40	170.00	16.350*	0.820	0.805	1.00*	175.00
24	70.40	199.90	24.625*	0.820	0.805	1.00*	135.00
25	70.45	239.60	37.850*	0.820	0.805	2.50*	256.00
26	70.45	279.80	53.550*	0.820	0.805	2.50*	199.50
27	70.45	330.15	37.000**	0.820	0.805	2.50*	154.75

TABLE C-15

OBSERVED DATA FOR SAMPLE LS-3-A

600-2, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.95	1.70*	0.150	0.790	0.785	100.00	797.25
2	70.95	3.10*	0.150	0.790	0.785	100.00	427.75
3	70.95	5.10*	0.200	0.790	0.785	100.00	253.02
4	71.00	8.10*	0.250	0.790	0.795	100.00	153.23
5	71.00	11.00*	0.300	0.790	0.795	100.00	108.88
6	71.00	15.00*	0.350	0.790	0.795	100.00	76.10
7	71.00	20.10*	0.500	0.800	0.795	100.00	53.60
8	71.00	26.00*	0.650	0.800	0.795	100.00	39.00
9	71.00	34.15*	0.900	0.800	0.795	100.00	27.40
10	71.00	44.10*	1.350	0.800	0.795	100.00	19.37
11	71.00	56.15*	1.950	0.805	0.800	100.00	13.80
12	71.00	34.10	4.500	0.810	0.802	1.00*	2958.00
13	71.00	44.00	6.550	0.810	0.802	1.00*	1943.00
14	71.00	56.00	10.200	0.815	0.810	1.00*	1326.75
15	71.00	70.05	16.500	0.815	0.810	1.00*	920.71
16	71.00	86.10	25.700	0.850	0.840	1.00*	653.00
17	71.00	104.00	3.100*	0.850	0.840	1.00*	474.50
18	71.00	124.00	5.200*	0.850	0.840	1.00*	351.47
19	71.00	144.00	8.000*	0.850	0.840	1.00*	272.00
20	71.00	170.00	12.650*	0.850	0.840	1.00*	206.00
21	71.00	200.00	19.350*	0.850	0.840	1.00*	158.50
22	71.00	239.60	30.400*	0.855	0.845	2.50*	300.00
23	71.00	279.80	43.800*	0.855	0.845	2.50*	232.50
24	71.00	330.00	30.500**	0.855	0.845	2.50*	178.50
25	71.00	399.00	45.750**	0.855	0.845	2.50*	133.00

TABLE C-16

OBSERVED DATA FOR SAMPLE LS-3-A

800-3, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.60	1.85*	0.150	0.855	0.860	100.00	788.94
2	70.60	4.90*	0.200	0.855	0.860	100.00	285.25
3	70.55	8.15*	0.250	0.865	0.860	100.00	164.61
4	70.55	11.20*	0.250	0.865	0.860	100.00	115.39
5	70.50	15.21*	0.350	0.870	0.870	100.00	81.18
6	70.50	20.10*	0.450	0.870	0.870	100.00	58.17
7	70.50	26.00*	0.550	0.870	0.870	100.00	42.30
8	70.50	34.00*	0.800	0.870	0.870	100.00	29.92
9	70.50	44.00*	1.150	0.870	0.870	100.00	21.16
10	70.45	56.25*	1.750	0.878	0.877	100.00	14.99
11	70.40	34.20	5.050	0.885	0.875	1.00*	3112.82
12	70.37	44.10	6.750	0.885	0.880	1.00*	2114.60
13	70.37	56.00	9.800	0.885	0.880	1.00*	1451.10
14	70.37	70.00	15.150	0.885	0.880	1.00*	1008.61
15	70.35	86.00	25.750	0.890	0.885	1.00*	714.81
16	70.35	104.20	2.825*	0.890	0.885	1.00*	517.67
17	70.35	124.00	4.550*	0.890	0.885	1.00*	384.61
18	70.35	144.00	6.950*	0.890	0.885	1.00*	298.13
19	70.35	170.00	11.000*	0.890	0.885	1.00*	225.65
20	70.35	199.80	16.900*	0.895	0.885	1.00*	171.50
21	70.35	239.80	26.925*	0.895	0.885	1.00*	127.20
22	70.35	280.00	38.850*	0.895	0.885	1.00*	99.56
23	70.35	329.90	56.400*	0.895	0.885	1.00*	77.20
24	70.35	398.80	41.000*	0.895	0.885	1.00*	56.90

TABLE C-17

OBSERVED DATA FOR SAMPLE LS-3-A

1000-4, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.20	2.00*	0.050	0.865	0.865	100.00	780.67
2	70.20	5.05*	0.100	0.865	0.865	100.00	297.00
3	70.15	8.10*	0.150	0.885	0.885	100.00	178.00
4	70.15	10.90*	0.200	0.885	0.885	100.00	128.00
5	70.15	15.14*	0.250	0.893	0.892	100.00	87.79
6	70.15	20.00*	0.350	0.893	0.892	100.00	63.00
7	70.15	29.90*	0.600	0.893	0.892	100.00	38.06
8	70.10	40.00*	1.000	0.900	0.900	100.00	25.94
9	70.10	56.10*	1.500	0.900	0.900	100.00	16.18
10	70.10	36.10	4.600	0.900	0.900	1.00*	3085.11
11	70.15	46.20	6.250	0.873	0.867	1.00*	2115.04
12	70.15	58.10	9.200	0.873	0.867	1.00*	1468.48
13	70.15	72.10	14.000	0.890	0.880	1.00*	1029.00
14	70.15	88.00	21.800	0.890	0.880	1.00*	736.00
15	70.15	105.90	2.550*	0.890	0.880	1.00*	537.00
16	70.17	126.10	4.100*	0.890	0.885	1.00*	400.70
17	70.17	150.00	6.725*	0.890	0.885	1.00*	300.00
18	70.17	176.00	10.450*	0.900	0.885	1.00*	229.50
19	70.20	205.85	15.950*	0.900	0.885	1.00*	177.00
20	70.20	240.00	23.700*	0.900	0.885	1.00*	138.40
21	70.20	280.00	34.400*	0.900	0.885	1.00*	107.63
22	70.20	329.80	50.300*	0.900	0.885	2.50*	207.75
23	70.20	397.00	36.500**	0.900	0.885	2.50*	154.70

TABLE C-18

OBSERVED DATA FOR SAMPLE SS-1-A

400-1, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.20	50.05	0.00	0.830	0.830	100.00	1253.40
2	70.20	54.00	0.00	0.855	0.865	100.00	1133.50
3	70.20	60.00	0.00	0.870	0.870	100.00	986.20
4	70.20	68.00	0.00	0.870	0.870	100.00	830.75
5	70.20	78.00	0.00	0.870	0.870	100.00	682.65
6	70.20	88.00	0.00	0.870	0.870	100.00	572.50
7	70.20	100.00	0.00	0.870	0.865	100.00	473.00
8	70.20	114.00	0.025	0.870	0.865	100.00	390.40
9	70.15	130.10	0.050	0.870	0.845	100.00	315.50
10	70.15	150.00	0.100	0.870	0.845	100.00	252.25
11	70.15	170.00	0.123	0.870	0.845	100.00	207.00
12	70.15	189.80	0.150	0.870	0.845	100.00	172.00
13	70.15	219.80	0.175	0.875	0.865	100.00	134.00
14	70.15	249.85	0.200	0.875	0.865	100.00	107.50
15	70.15	279.75	0.250	0.875	0.865	100.00	88.30
16	70.15	309.90	0.300	0.875	0.865	100.00	72.50
17	70.15	349.90	0.400	0.880	0.875	100.00	55.60
18	70.15	389.60	0.500	0.880	0.875	100.00	44.70
19	70.15	440.00	0.650	0.880	0.875	100.00	33.80
20	70.15	522.00	1.400	0.880	0.875	100.00	18.00

TABLE C-19

OBSERVED DATA FOR SAMPLE SS-1-A

600-2, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.025	60.10	0.050	0.865	0.870	100.00	1299.20
2	70.125	68.00	0.050	0.890	0.880	100.00	1169.00
3	70.700	68.00	0.100	0.830	0.815	100.00	1164.00
4	70.700	80.00	0.100	0.830	0.825	100.00	931.00
5	70.650	94.00	0.100	0.835	0.800	100.00	740.50
6	70.650	110.20	0.100	0.833	0.817	100.00	586.00
7	70.650	129.80	0.100	0.833	0.817	100.00	458.00
8	70.620	150.00	0.100	0.835	0.825	100.00	367.00
9	70.620	180.00	0.150	0.835	0.825	100.00	278.00
10	70.600	209.90	0.150	0.835	0.825	100.00	215.50
11	70.600	249.90	0.200	0.835	0.825	100.00	161.60
12	70.600	290.00	0.200	0.835	0.825	100.00	125.50
13	70.600	339.90	0.300	0.835	0.820	100.00	96.00
14	70.600	390.20	0.350	0.835	0.820	100.00	75.20
15	70.600	449.50	0.400	0.835	0.820	100.00	59.00
16	70.600	521.50	0.500	0.840	0.820	100.00	44.80
17	70.600	622.00	0.725	0.840	0.820	100.00	32.00
18	70.600	767.00	1.150	0.840	0.820	100.00	21.00
19	70.600	875.00	1.600	0.845	0.825	100.00	16.00
20	70.600	1021.00	5.100	0.845	0.825	1.00*	3130.00
21	70.600	1158.50	6.700	0.845	0.825	1.00*	2140.00
22	70.550	1254.00	9.200	0.845	0.825	1.00*	1525.00

TABLE C-20

OBSERVED DATA FOR SAMPLE SS-1-A

1000-3, N_2

1. (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.460	80.00	0.050	0.890	0.880	100.00	1050.00
2	70.460	89.80	0.050	0.905	0.900	100.00	888.00
3	70.500	103.80	0.075	0.915	0.910	100.00	723.00
4	70.500	120.10	0.100	0.915	0.910	100.00	585.00
5	70.500	140.00	0.100	0.915	0.910	100.00	465.00
6	70.500	160.00	0.125	0.915	0.910	100.00	379.00
7	70.500	189.85	0.150	0.915	0.910	100.00	293.00
8	70.500	230.15	0.150	0.915	0.910	100.00	215.50
9	70.525	269.40	0.175	0.920	0.910	100.00	164.50
10	70.650	321.60	0.200	0.940	0.905	100.00	128.20
11	70.650	379.30	0.250	0.940	0.905	100.00	95.80
12	70.650	488.00	0.375	0.940	0.925	100.00	61.60
13	70.650	599.00	0.550	0.940	0.925	100.00	43.40
14	70.650	793.00	0.900	0.940	0.925	100.00	26.00
15	70.700	1002.00	1.500	0.955	0.935	100.00	16.50
16	70.700	1209.00	4.200	0.955	0.935	1.00*	3240.00
17	70.700	1413.00	5.600	0.955	0.935	1.00*	2300.00
18	70.700	1609.00	7.800	0.955	0.935	1.00*	1650.00
19	70.700	1917.00	14.700	0.955	0.935	1.00*	980.00

TABLE C-21

OBSERVED DATA FOR SAMPLE SS-2-A

200-1A, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.500	3.00*	0.000	0.900	0.900	100.00	644.50
2	70.500	4.00*	0.000	0.908	0.902	100.00	481.00
3	70.500	5.00*	0.000	0.908	0.902	100.00	383.30
4	70.500	6.00*	0.050	0.908	0.902	100.00	312.80
5	70.450	7.10*	0.100	0.910	0.905	100.00	261.80
6	70.450	8.05*	0.125	0.910	0.905	100.00	230.00
7	70.450	10.00*	0.150	0.910	0.905	100.00	180.00
8	70.450	12.00*	0.175	0.910	0.905	100.00	146.80
9	70.450	15.00*	0.200	0.910	0.905	100.00	114.50
10	70.450	18.00*	0.250	0.910	0.905	100.00	93.60
11	70.450	24.00*	0.350	0.910	0.905	100.00	65.60
12	70.450	30.00*	0.450	0.910	0.905	100.00	49.80
13	70.350	38.00*	0.600	0.910	0.905	100.00	36.50
14	70.350	48.00*	0.900	0.910	0.905	100.00	26.85
15	70.350	60.00*	1.200	0.910	0.905	100.00	19.70
16	70.350	36.00	4.700	0.910	0.905	1.00*	4134.50
17	70.275	44.10	5.500	0.915	0.905	1.00*	3113.15
18	70.275	54.00	6.850	0.915	0.905	1.00*	2284.04
19	70.275	66.00	8.900	0.915	0.905	1.00*	1668.00
20	70.275	80.00	12.350	0.915	0.905	1.00*	1220.00
21	70.250	94.00	16.900	0.915	0.905	1.00*	934.00
22	70.250	110.00	1.750*	0.915	0.905	1.00*	715.00
23	70.250	130.10	2.700*	0.915	0.905	1.00*	538.53
24	70.250	150.00	3.900*	0.915	0.905	1.00*	420.00
25	70.250	170.00	5.700*	0.915	0.905	1.00*	337.50
26	70.250	200.00	9.050*	0.915	0.905	1.00*	254.00
27	70.200	229.70	13.400*	0.920	0.905	1.00*	199.50
28	70.200	269.80	20.900*	0.920	0.905	1.00*	151.50
29	70.150	311.20	29.500*	0.940	0.930	1.00*	119.40
30	70.150	361.00	42.700*	0.940	0.930	1.00*	93.08

TABLE C-22

OBSERVED DATA FOR SAMPLE SS-2-A

200-1B, Ar

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.250	2.00*	0.000	0.870	0.845	100.00	1276.00
2	70.250	4.00*	0.075	0.890	0.880	100.00	617.70
3	70.250	5.10*	0.100	0.890	0.880	100.00	484.00
4	70.250	8.05*	0.180	0.890	0.880	100.00	297.00
5	70.300	10.10*	0.200	0.890	0.885	100.00	231.40
6	70.300	12.10*	0.225	0.890	0.885	100.00	189.70
7	70.300	15.00*	0.250	0.890	0.885	100.00	148.80
8	70.300	18.00*	0.300	0.890	0.885	100.00	120.60
9	70.350	23.10*	0.400	0.895	0.890	100.00	89.72
10	70.350	30.10*	0.500	0.895	0.890	100.00	65.00
11	70.350	40.10*	0.700	0.895	0.890	100.00	44.80
12	70.350	52.20*	1.000	0.895	0.890	100.00	31.58
13	70.350	32.10	1.400	0.900	0.890	100.00	23.06
14	70.350	40.10	2.000	0.900	0.890	100.00	16.65
15	70.350	50.00	3.000	0.900	0.890	100.00	11.97
16	70.350	60.00	6.200	0.900	0.890	1.00**	2545.00
17	70.350	74.00	9.000	0.900	0.890	1.00**	1815.50
18	70.350	90.00	13.200	0.900	0.890	1.00**	1313.50
19	70.400	110.10	20.400	0.898	0.887	1.00**	936.00
20	70.400	130.00	2.250*	0.898	0.887	1.00**	702.50
21	70.400	150.00	3.300*	0.898	0.887	1.00**	549.00
22	70.400	170.00	4.750*	0.898	0.887	1.00**	442.00
23	70.400	199.90	7.700*	0.898	0.887	1.00**	332.00
24	70.400	230.00	11.500*	0.898	0.887	1.00**	259.50
25	70.400	270.00	18.100*	0.898	0.887	1.00**	196.50
26	70.400	309.80	26.250*	0.910	0.890	1.00*	155.63
27	70.400	359.70	38.350*	0.910	0.890	2.50*	301.00
28	70.400	440.00	30.500**	0.910	0.890	2.50*	211.50
29	70.400	532.00	46.600**	0.910	0.890	2.50*	153.00

TABLE C-23

OBSERVED DATA FOR SAMPLE SS-2-A

200-1C, CH_4

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.150	2.00*	0.000	0.920	0.905	100.00	692.00
2	70.150	3.05*	0.000	0.940	0.935	100.00	446.50
3	70.150	4.00*	0.000	0.920	0.905	100.00	337.50
4	70.150	5.00*	0.000	0.920	0.905	100.00	266.70
5	69.850	6.00*	0.000	0.860	0.855	100.00	221.60
6	69.850	8.00*	0.050	0.860	0.855	100.00	160.70
7	69.850	10.00*	0.100	0.860	0.855	100.00	125.10
8	69.900	12.00*	0.150	0.878	0.872	100.00	101.50
9	69.900	15.00*	0.200	0.878	0.872	100.00	78.20
10	69.900	18.00*	0.250	0.878	0.872	100.00	62.90
11	69.900	23.00*	0.300	0.878	0.872	100.00	46.65
12	69.950	30.00*	0.400	0.885	0.880	100.00	33.60
13	69.950	39.95*	0.600	0.885	0.880	100.00	23.00
14	69.950	52.00*	1.000	0.885	0.880	100.00	16.00
15	69.950	32.10	3.800	0.890	0.880	1.00*	3286.89
16	69.950	40.05	4.500	0.890	0.880	1.00*	2359.84
17	69.950	50.00	5.900	0.890	0.880	1.00*	1680.69
18	69.950	60.00	7.600	0.890	0.880	1.00*	1256.16
19	69.950	74.00	11.000	0.890	0.880	1.00*	893.48
20	70.050	90.00	16.200	0.885	0.875	1.00*	645.01
21	70.000	110.10	2.000*	0.870	0.860	1.00*	459.77
22	70.000	130.00	3.200*	0.870	0.860	1.00*	347.73
23	70.000	150.00	4.850*	0.870	0.860	1.00*	272.52
24	70.000	169.70	7.000*	0.870	0.860	1.00*	221.50
25	70.000	199.90	11.300*	0.890	0.870	1.00*	167.00
26	70.000	229.90	16.700*	0.890	0.870	1.00*	132.00
27	70.000	269.40	25.650*	0.890	0.870	2.50*	253.50
28	70.000	310.25	36.900*	0.890	0.870	2.50*	198.00

TABLE C-24

OBSERVED DATA FOR SAMPLE SS-2-A

400-2, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.60	2.00*	0.150	0.925	0.920	100.00	1103.00
2	70.60	3.00*	0.150	0.925	0.920	100.00	729.50
3	70.60	4.00*	0.150	0.925	0.915	100.00	538.50
4	70.60	5.00*	0.150	0.925	0.915	100.00	426.20
5	70.60	6.00*	0.150	0.925	0.915	100.00	352.50
6	70.60	8.00*	0.150	0.925	0.915	100.00	257.80
7	70.60	10.00*	0.175	0.925	0.915	100.00	201.60
8	70.60	12.00*	0.200	0.925	0.915	100.00	164.60
9	70.60	15.00*	0.250	0.920	0.910	100.00	128.10
10	70.60	18.00*	0.275	0.920	0.910	100.00	104.00
11	70.60	24.10*	0.350	0.920	0.910	100.00	73.25
12	70.60	32.10*	0.450	0.920	0.910	100.00	51.20
13	70.60	42.10*	0.650	0.915	0.905	100.00	36.30
14	70.60	54.00*	0.950	0.915	0.905	100.00	25.80
15	70.60	34.00	1.400	0.915	0.905	100.00	18.00
16	70.60	42.10	2.050	0.865	0.860	100.00	13.20
17	70.00	42.10	2.000	0.890	0.885	100.00	13.20
18	70.60	52.00	7.250	0.885	0.885	1.00*	2726.92
19	70.60	64.00	8.900	0.885	0.885	1.00*	1965.99
20	70.60	78.00	11.600	0.900	0.895	1.00*	1430.22
21	70.60	94.00	15.800	0.900	0.895	1.00*	1048.21
22	70.60	112.10	22.200	0.900	0.895	1.00*	774.60
23	70.60	132.00	2.350*	0.900	0.895	1.00*	585.09
24	70.60	154.00	3.500*	0.900	0.895	1.00*	447.86
25	70.60	176.10	5.100*	0.905	0.900	1.00*	355.38
26	70.60	199.80	7.400*	0.905	0.900	1.00*	286.00
27	70.60	229.90	11.050*	0.905	0.900	1.00*	224.00
28	70.60	259.70	15.600*	0.910	0.900	1.00*	181.50
29	70.60	300.10	23.000*	0.910	0.900	1.00*	142.00
30	70.60	350.00	34.150*	0.910	0.900	1.00*	109.49
31	70.60	439.00	29.200**	0.910	0.900	2.50*	186.50
32	70.60	439.00	28.800**	0.910	0.900	2.50*	186.50

TABLE C-25

OBSERVED DATA FOR SAMPLE SS-2-A

600-3, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.85	2.000*	0.050	0.875	0.875	100.00	1138.00
2	70.85	3.00*	0.050	0.875	0.875	100.00	744.80
3	70.80	4.00*	0.050	0.885	0.885	100.00	553.70
4	70.55	5.025*	0.050	0.865	0.855	100.00	434.50
5	70.55	6.000*	0.050	0.865	0.855	100.00	362.50
6	70.55	8.000*	0.750	0.850	0.880	100.00	265.70
7	70.55	10.000*	0.100	0.860	0.865	100.00	208.60
8	70.55	13.000*	0.150	0.860	0.865	100.00	155.00
9	70.50	16.000*	0.175	0.860	0.865	100.00	122.50
10	70.50	20.000*	0.225	0.860	0.865	100.00	94.55
11	70.50	24.000*	0.275	0.860	0.865	100.00	75.99
12	70.50	30.000*	0.350	0.860	0.860	100.00	57.69
13	70.50	36.000*	0.450	0.860	0.860	100.00	45.80
14	70.50	44.000*	0.600	0.860	0.860	100.00	35.30
15	70.50	54.000*	0.800	0.860	0.860	100.00	26.74
16	70.50	32.00	1.150	0.860	0.860	100.00	20.50
17	70.45	40.10	5.500	0.860	0.860	1.00*	4206.30
18	70.45	50.00	6.500	0.905	0.905	1.00*	3026.63
19	70.23	62.00	7.500	0.830	0.825	1.00*	2148.76
20	70.23	76.00	9.950	0.830	0.825	1.00*	1553.13
21	70.23	92.00	13.700	0.840	0.835	1.00*	1134.00
22	70.23	110.20	19.600	0.840	0.835	1.00*	835.80
23	70.23	130.00	28.800	0.850	0.840	1.00*	629.00
24	70.23	150.10	3.050*	0.850	0.840	1.00*	490.00
25	70.23	180.00	5.125*	0.850	0.840	1.00*	357.00
26	70.23	210.20	8.100*	0.855	0.850	1.00*	271.50
27	70.23	249.90	13.200*	0.855	0.850	1.00*	200.00
28	70.15	299.70	21.600*	0.905	0.915	1.00*	146.50
29	70.23	339.80	30.150*	0.865	0.855	1.00*	118.85
30	70.23	399.90	44.900*	0.865	0.855	1.00*	90.20

TABLE C-26

OBSERVED DATA FOR SAMPLE SS-2-A

800-4, N_2

1 (ND)	2 (P_b)	3 (P_e)	4 (P_w)	5 (TC_e)	6 (TC_w)	7 (V)	8 (t_m)
1	70.03	3.05*	0.150	0.930	0.930	100.00	753.00
2	70.03	4.90*	0.150	0.930	0.930	100.00	457.20
3	70.03	6.90*	0.150	0.930	0.930	100.00	318.00
4	70.03	8.00*	0.150	0.930	0.930	100.00	271.75
5	69.95	11.00*	0.175	0.930	0.930	100.00	191.85
6	69.95	15.00*	0.225	0.930	0.930	100.00	135.00
7	69.95	20.10*	0.300	0.930	0.930	100.00	96.00
8	69.95	26.00*	0.350	0.930	0.930	100.00	70.30
9	69.95	34.05*	0.450	0.930	0.930	100.00	50.20
10	69.95	44.00*	0.650	0.930	0.930	100.00	36.20
11	69.95	54.05*	0.850	0.930	0.930	100.00	27.40
12	69.95	34.00	1.300	0.925	0.930	100.00	18.82
13	69.95	44.00	2.000	0.925	0.930	100.00	13.20
14	69.90	55.95	6.200	0.940	0.940	1.00*	2602.94
15	69.90	70.00	8.200	0.940	0.940	1.00*	1820.55
16	69.90	86.00	11.400	0.940	0.940	1.00*	1303.00
17	69.85	104.00	16.400	0.905	0.900	1.00*	942.00
18	69.85	134.00	2.150*	0.905	0.900	1.00*	608.50
19	69.85	160.00	3.450*	0.905	0.900	1.00*	447.00
20	69.84	189.95	5.650*	0.920	0.915	1.00*	331.00
21	69.84	229.70	9.900*	0.920	0.915	1.00*	236.50
22	69.84	269.70	15.600*	0.920	0.915	1.00*	178.00
23	69.84	320.00	24.770*	0.925	0.925	1.00*	133.00
24	69.84	319.90	24.700*	0.925	0.925	2.50*	338.50
25	69.84	379.70	38.150*	0.925	0.925	2.50*	251.50
26	69.84	449.00	27.850**	0.925	0.925	2.50*	188.35

APPENDIX D

TABULATED PROCESSED DATA AND RESULTS OF PARAMETER ESTIMATION TECHNIQUES

TABLE D-1

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 600 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-1
RUN NUMBER = 1A

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.145173E 02	0.135841E 02	0.453449E-03	0.176074E-01	0.932889E 01
0.148120E 02	0.135841E 02	0.600970E-03	0.176151E-01	0.933829E 01
0.153031E 02	0.135841E 02	0.850510E-03	0.176154E-01	0.933840E 01
0.157943E 02	0.135841E 02	0.111202E-02	0.176264E-01	0.935173E 01
0.163837E 02	0.135841E 02	0.142384E-02	0.176267E-01	0.935187E 01
0.170713E 02	0.135877E 02	0.179367E-02	0.176271E-01	0.935202E 01
0.178572E 02	0.135895E 02	0.225998E-02	0.176336E-01	0.935967E 01
0.189675E 02	0.135720E 02	0.293815E-02	0.176395E-01	0.936646E 01
0.199498E 02	0.135756E 02	0.353372E-02	0.176385E-01	0.936482E 01
0.211777E 02	0.135774E 02	0.433380E-02	0.176391E-01	0.936510E 01
0.228968E 02	0.135828E 02	0.556828E-02	0.176401E-01	0.936551E 01
0.253894E 02	0.135882E 02	0.736979E-02	0.176414E-01	0.936608E 01
0.282995E 02	0.135973E 02	0.977411E-02	0.176430E-01	0.936676E 01
0.317376E 02	0.136045E 02	0.128186E-01	0.176464E-01	0.936942E 01
0.356625E 02	0.135971E 02	0.167510E-01	0.176628E-01	0.938794E 01
0.401566E 02	0.137488E 02	0.213948E-01	0.176653E-01	0.938901E 01
0.455857E 02	0.137813E 02	0.276338E-01	0.176795E-01	0.940417E 01
0.495357E 02	0.138059E 02	0.326193E-01	0.176817E-01	0.940510E 01
0.535357E 02	0.138427E 02	0.377525E-01	0.176893E-01	0.941275E 01
0.585357E 02	0.138796E 02	0.446401E-01	0.176921E-01	0.941393E 01
0.655108E 02	0.139532E 02	0.549054E-01	0.176959E-01	0.941559E 01
0.735358E 02	0.140515E 02	0.677311E-01	0.176836E-01	0.939687E 01
0.835388E 02	0.141742E 02	0.849688E-01	0.176891E-01	0.939924E 01
0.955108E 02	0.144444E 02	0.107290E 00	0.176958E-01	0.940211E 01
0.107535E 03	0.147391E 02	0.131023E 00	0.177025E-01	0.940501E 01
0.133535E 03	0.156232E 02	0.187784E 00	0.177256E-01	0.942184E 01
0.163535E 03	0.171212E 02	0.261760E 00	0.177429E-01	0.942931E 01
0.203535E 03	0.199454E 02	0.367578E 00	0.177664E-01	0.943954E 01
0.253335E 03	0.244149E 02	0.512734E 00	0.178073E-01	0.946635E 01
0.313935E 03	0.310701E 02	0.707171E 00	0.178447E-01	0.948282E 01
0.412935E 03	0.441357E 02	0.100759E 01	0.179107E-01	0.951450E 01

TABLE D-2

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.1183E 02	0.4356E 01	0.1395E 10
1	0.6251E 01	0.8633E 01	0.2791E 10
2	0.7546E 01	0.9372E 01	0.3116E 10
3	0.7792E 01	0.9175E 01	0.3084E 10
4	0.7797E 01	0.9183E 01	0.3084E 10
5	0.7797E 01	0.9183E 01	0.3084E 10

FRACTIONAL POROSITY = 0.9718E-01

SUM OF ERROR SQUARES = 0.1603E 02
VARIANCE = 0.5725E 00
STANDARD DEVIATION = 0.7566E 00

VARIANCE-COVARIANCE MATRIX

0.17610E-01	-0.10538E 00	0.57328E 07
-0.10538E 00	0.69509E 00	-0.31417E 08
0.57328E 07	-0.31417E 08	0.22994E 16

CORRELATION MATRIX

1.00000	-0.95247	0.90088
-0.95247	1.00000	-0.78583
-0.90088	-0.78583	1.00000

TABLE D-3

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14517E 02	0.14394E 02	0.84783E 00
0.14812E 02	0.14655E 02	0.10547E 01
0.15303E 02	0.15090E 02	0.13903E 01
0.15794E 02	0.15551E 02	0.15369E 01
0.16383E 02	0.16078E 02	0.18622E 01
0.17071E 02	0.16694E 02	0.22103E 01
0.17857E 02	0.17483E 02	0.20920E 01
0.18967E 02	0.18565E 02	0.21206E 01
0.19949E 02	0.19481E 02	0.23477E 01
0.21177E 02	0.20689E 02	0.23042E 01
0.22896E 02	0.22505E 02	0.17070E 01
0.25389E 02	0.24933E 02	0.17977E 01
0.28299E 02	0.27998E 02	0.10641E 01
0.31737E 02	0.31549E 02	0.59384E 00
0.35662E 02	0.35806E 02	-0.40270E 00
0.40156E 02	0.40391E 02	-0.58446E 00
0.45585E 02	0.45974E 02	-0.85169E 00
0.49535E 02	0.50084E 02	-0.11073E 01
0.53535E 02	0.54069E 02	-0.99775E 00
0.58535E 02	0.59083E 02	-0.93626E 00
0.65510E 02	0.66037E 02	-0.80418E 00
0.73535E 02	0.73975E 02	-0.59789E 00
0.83538E 02	0.83894E 02	-0.42637E 00
0.95510E 02	0.95727E 02	-0.22666E 00
0.10753E 03	0.10737E 03	0.15232E 00
0.13353E 03	0.13284E 03	0.51408E 00
0.16353E 03	0.16281E 03	0.43864E 00
0.20353E 03	0.20218E 03	0.66248E 00
0.25333E 03	0.25275E 03	0.23069E 00
0.31393E 03	0.31681E 03	-0.91719E 00
0.41293E 03	0.41184E 03	0.26368E 00

TABLE D-4

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 600 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = ARGON
SAMPLE NUMBER = LS-1
RUN NUMBER = 18

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.146364E 02	0.135067E 02	0.438630E-03	0.223493E-01	0.118207E 02
0.149802E 02	0.135067E 02	0.572430E-03	0.223574E-01	0.118302E 02
0.155205E 02	0.135067E 02	0.805890E-03	0.223578E-01	0.118303E 02
0.161099E 02	0.135067E 02	0.105092E-02	0.223582E-01	0.118305E 02
0.168221E 02	0.135067E 02	0.136281E-02	0.223622E-01	0.118348E 02
0.175834E 02	0.135067E 02	0.170987E-02	0.223628E-01	0.118350E 02
0.184429E 02	0.135067E 02	0.211760E-02	0.223634E-01	0.118352E 02
0.196611E 02	0.135007E 02	0.272368E-02	0.223642E-01	0.118354E 02
0.208399E 02	0.135064E 02	0.333444E-02	0.223650E-01	0.118357E 02
0.223723E 02	0.135097E 02	0.416805E-02	0.223661E-01	0.118361E 02
0.234133E 02	0.135345E 02	0.483282E-02	0.222890E-01	0.117444E 02
0.259182E 02	0.135381E 02	0.634184E-02	0.222995E-01	0.117553E 02
0.283003E 02	0.135453E 02	0.789637E-02	0.223100E-01	0.117663E 02
0.322541E 02	0.135579E 02	0.107091E-01	0.223181E-01	0.117733E 02
0.370920E 02	0.136175E 02	0.145356E-01	0.223294E-01	0.117838E 02
0.424702E 02	0.136970E 02	0.191847E-01	0.223332E-01	0.117850E 02
0.454664E 02	0.137129E 02	0.219619E-01	0.223493E-01	0.118023E 02
0.495664E 02	0.137571E 02	0.260300E-01	0.223554E-01	0.118070E 02
0.536584E 02	0.136450E 02	0.304833E-01	0.223047E-01	0.117438E 02
0.584584E 02	0.136941E 02	0.360070E-01	0.223152E-01	0.117533E 02
0.654584E 02	0.137531E 02	0.439390E-01	0.223095E-01	0.117418E 02
0.734584E 02	0.138513E 02	0.544883E-01	0.223302E-01	0.117616E 02
0.834584E 02	0.139987E 02	0.684872E-01	0.223374E-01	0.117636E 02
0.954584E 02	0.142050E 02	0.869096E-01	0.223495E-01	0.117704E 02
0.107458E 03	0.145389E 02	0.106599E 00	0.223716E-01	0.117893E 02
0.133458E 03	0.154721E 02	0.154568E 00	0.223908E-01	0.117952E 02
0.163458E 03	0.169702E 02	0.213900E 00	0.224135E-01	0.118022E 02
0.203458E 03	0.196716E 02	0.303359E 00	0.224445E-01	0.118120E 02
0.253458E 03	0.240429E 02	0.426572E 00	0.225075E-01	0.118553E 02
0.313658E 03	0.305262E 02	0.581115E 00	0.225572E-01	0.118724E 02

TABLE D-5

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.1183E 02	0.4356E 01	0.1395E 10
1	0.6281E 01	0.8705E 01	0.2704E 10
2	0.7569E 01	0.9433E 01	0.3005E 10
3	0.7798E 01	0.9299E 01	0.2965E 10
4	0.7802E 01	0.9304E 01	0.2965E 10
5	0.7802E 01	0.9304E 01	0.2965E 10

FRACTIONAL POROSITY = 0.9863E-01

SUM OF ERROR SQUARES = 0.3707E 01
VARIANCE = 0.1373E 00
STANDARD DEVIATION = 0.3705E 00

VARIANCE-COVARIANCE MATRIX

0.60698E-02	-0.34166E-01	0.29300E 07
-0.34166E-01	0.20883E 00	-0.15299E 08
0.29300E 07	-0.15299E 08	0.16892E 16

CORRELATION MATRIX

1.00000	-0.95963	0.91502
-0.95963	1.00000	-0.81456
0.91502	-0.81456	1.00000

TABLE D-6

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14636E 02	0.14497E 02	0.95079E 00
0.14980E 02	0.14788E 02	0.12785E 01
0.15520E 02	0.15318E 02	0.13026E 01
0.16109E 02	0.15842E 02	0.16598E 01
0.16822E 02	0.16509E 02	0.18593E 01
0.17583E 02	0.17237E 02	0.19684E 01
0.18442E 02	0.18074E 02	0.20003E 01
0.19661E 02	0.19274E 02	0.19687E 01
0.20839E 02	0.20449E 02	0.18723E 01
0.22372E 02	0.21983E 02	0.17361E 01
0.23413E 02	0.23141E 02	0.11619E 01
0.25918E 02	0.25655E 02	0.10127E 01
0.28300E 02	0.28094E 02	0.72691E 00
0.32254E 02	0.32156E 02	0.30319E 00
0.37092E 02	0.37182E 02	-0.24460E 00
0.42470E 02	0.42650E 02	-0.42319E 00
0.45466E 02	0.45696E 02	-0.50639E 00
0.49566E 02	0.49884E 02	-0.64231E 00
0.53658E 02	0.53958E 02	-0.55852E 00
0.58458E 02	0.58981E 02	-0.89506E 00
0.65458E 02	0.65530E 02	-0.11042E 00
0.73458E 02	0.73767E 02	-0.42131E 00
0.83458E 02	0.83699E 02	-0.28869E 00
0.95458E 02	0.95694E 02	-0.24768E 00
0.10745E 03	0.10760E 03	-0.13261E 00
0.13345E 03	0.13370E 03	-0.18410E 00
0.16345E 03	0.16263E 03	0.50085E 00
0.20345E 03	0.20253E 03	0.45295E 00
0.25345E 03	0.25371E 03	-0.10116E 00
0.31365E 03	0.31397E 03	-0.99689E-01

TABLE D-7

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 600 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = METHANE
SAMPLE NUMBER = LS-1
RUN NUMBER = 1C

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.142241E 02	0.134874E 02	0.548390E-03	0.110645E-01	0.586259E 01
0.146171E 02	0.134874E 02	0.835960E-03	0.110645E-01	0.586246E 01
0.150223E 02	0.134874E 02	0.117177E-02	0.110645E-01	0.586232E 01
0.154766E 02	0.134874E 02	0.153611E-02	0.110645E-01	0.586216E 01
0.161888E 02	0.134874E 02	0.210919E-02	0.110645E-01	0.586191E 01
0.168027E 02	0.134874E 02	0.263649E-02	0.110645E-01	0.586170E 01
0.175640E 02	0.134874E 02	0.330355E-02	0.110645E-01	0.586144E 01
0.184481E 02	0.134874E 02	0.415447E-02	0.110646E-01	0.586113E 01
0.196514E 02	0.134910E 02	0.533454E-02	0.110646E-01	0.586071E 01
0.208057E 02	0.134928E 02	0.645165E-02	0.110646E-01	0.586031E 01
0.223774E 02	0.134982E 02	0.816057E-02	0.110646E-01	0.585977E 01
0.238509E 02	0.135018E 02	0.979268E-02	0.110647E-01	0.585926E 01
0.261839E 02	0.135091E 02	0.125778E-01	0.110647E-01	0.585845E 01
0.284971E 02	0.135830E 02	0.154385E-01	0.110612E-01	0.585322E 01
0.314971E 02	0.136149E 02	0.194578E-01	0.110648E-01	0.585657E 01
0.344971E 02	0.136321E 02	0.237678E-01	0.110649E-01	0.585553E 01
0.384971E 02	0.136444E 02	0.299760E-01	0.110650E-01	0.585414E 01
0.436971E 02	0.136935E 02	0.387603E-01	0.110615E-01	0.584792E 01
0.484971E 02	0.137328E 02	0.475836E-01	0.110616E-01	0.584625E 01
0.534971E 02	0.137525E 02	0.574503E-01	0.110653E-01	0.584892E 01
0.584971E 02	0.138360E 02	0.677563E-01	0.110654E-01	0.584716E 01
0.654971E 02	0.138998E 02	0.837640E-01	0.110655E-01	0.584472E 01
0.734971E 02	0.140177E 02	0.102614E 00	0.110657E-01	0.584192E 01
0.834971E 02	0.141970E 02	0.128855E 00	0.110659E-01	0.583841E 01
0.954971E 02	0.145039E 02	0.162861E 00	0.110678E-01	0.583613E 01
0.107697E 03	0.148969E 02	0.199192E 00	0.110680E-01	0.583179E 01
0.133497E 03	0.160634E 02	0.283248E 00	0.110706E-01	0.582497E 01
0.163497E 03	0.178929E 02	0.389892E 00	0.110713E-01	0.581403E 01
0.203497E 03	0.211714E 02	0.550956E 00	0.110702E-01	0.579672E 01
0.253497E 03	0.263900E 02	0.750578E 00	0.110715E-01	0.577782E 01
0.313697E 03	0.339539E 02	0.100204E 01	0.110730E-01	0.575472E 01

TABLE D-8

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.1183E 02	0.4356E 01	0.1395E 10
1	0.6523E 01	0.6896E 01	0.2846E 10
2	0.7710E 01	0.7449E 01	0.3173E 10
3	0.7902E 01	0.7349E 01	0.3145E 10
4	0.7905E 01	0.7352E 01	0.3145E 10
5	0.7905E 01	0.7352E 01	0.3145E 10

FRACTIONAL POROSITY = 0.1114E 00

SUM OF ERROR SQUARES = 0.2868E 01
VARIANCE = 0.1024E 00
STANDARD DEVIATION = 0.3200E 00

VARIANCE-COVARIANCE MATRIX

0.64062E-02	-0.32484E-01	0.20468E 07
-0.32484E-01	0.17763E 00	-0.95950E 07
0.20468E 07	-0.95950E 07	0.79629E 15

CORRELATION MATRIX

1.00000	-0.96295	0.90625
-0.96295	1.00000	-0.80675
0.90625	-0.80675	1.00000

TABLE D-9

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14224E 02	0.14174E 02	0.34861E 00
0.14617E 02	0.14518E 02	0.67138E 00
0.15022E 02	0.14937E 02	0.56264E 00
0.15476E 02	0.15372E 02	0.67195E 00
0.16188E 02	0.16034E 02	0.95237E 00
0.16802E 02	0.16639E 02	0.97140E 00
0.17564E 02	0.17382E 02	0.10357E 01
0.18448E 02	0.18322E 02	0.68212E 00
0.19651E 02	0.19566E 02	0.43198E 00
0.20805E 02	0.20671E 02	0.64549E 00
0.22377E 02	0.22327E 02	0.22486E 00
0.23850E 02	0.23813E 02	0.15699E 00
0.26183E 02	0.26218E 02	-0.13055E 00
0.28497E 02	0.28549E 02	-0.18331E 00
0.31497E 02	0.31590E 02	-0.29512E 00
0.34497E 02	0.34604E 02	-0.31176E 00
0.38497E 02	0.38628E 02	-0.34097E 00
0.43697E 02	0.43837E 02	-0.32047E 00
0.48497E 02	0.48656E 02	-0.32841E 00
0.53497E 02	0.53677E 02	-0.33793E 00
0.58497E 02	0.58580E 02	-0.14167E 00
0.65497E 02	0.65685E 02	-0.28832E 00
0.73497E 02	0.73420E 02	0.10463E 00
0.83497E 02	0.83433E 02	0.75638E-01
0.95497E 02	0.95463E 02	0.34784E-01
0.10769E 03	0.10743E 03	0.24604E 00
0.13349E 03	0.13300E 03	0.37188E 00
0.16349E 03	0.16276E 03	0.44842E 00
0.20349E 03	0.20468E 03	-0.58470E 00
0.25349E 03	0.25379E 03	-0.11604E 00
0.31369E 03	0.31327E 03	0.13547E 00

TABLE D-10

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 800 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE - NUMBER = LS-1
RUN NUMBER = 2

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MW*T*Z (CP*DEG R)
0.139496E 02	0.134584E 02	0.229800E-03	0.176692E-01	0.940529E 01
0.141460E 02	0.134584E 02	0.323090E-03	0.176734E-01	0.941047E 01
0.144407E 02	0.134584E 02	0.464950E-03	0.176736E-01	0.941054E 01
0.149319E 02	0.134584E 02	0.710360E-03	0.176486E-01	0.937951E 01
0.154230E 02	0.134584E 02	0.961840E-03	0.176415E-01	0.937060E 01
0.159239E 02	0.134681E 02	0.121934E-02	0.176744E-01	0.941089E 01
0.169062E 02	0.134681E 02	0.175037E-02	0.176882E-01	0.942748E 01
0.178885E 02	0.134735E 02	0.231820E-02	0.176887E-01	0.942771E 01
0.193620E 02	0.134771E 02	0.318854E-02	0.176896E-01	0.942806E 01
0.208354E 02	0.134789E 02	0.416215E-02	0.176904E-01	0.942841E 01
0.228001E 02	0.134825E 02	0.551059E-02	0.176838E-01	0.941951E 01
0.257470E 02	0.134915E 02	0.768486E-02	0.176854E-01	0.942020E 01
0.282318E 02	0.135260E 02	0.984768E-02	0.175655E-01	0.927138E 01
0.311788E 02	0.135368E 02	0.125385E-01	0.175671E-01	0.927204E 01
0.351081E 02	0.135476E 02	0.163457E-01	0.175692E-01	0.927292E 01
0.409529E 02	0.137174E 02	0.223740E-01	0.175725E-01	0.927427E 01
0.455874E 02	0.136790E 02	0.277845E-01	0.175766E-01	0.927732E 01
0.535374E 02	0.137330E 02	0.378733E-01	0.175850E-01	0.928410E 01
0.634874E 02	0.138189E 02	0.521510E-01	0.175905E-01	0.928636E 01
0.734874E 02	0.139466E 02	0.679046E-01	0.175960E-01	0.928864E 01
0.835124E 02	0.141013E 02	0.855086E-01	0.176015E-01	0.929094E 01
0.934874E 02	0.142782E 02	0.104005E 00	0.176102E-01	0.929714E 01
0.108537E 03	0.146662E 02	0.134069E 00	0.176186E-01	0.930065E 01
0.118687E 03	0.149854E 02	0.156164E 00	0.176243E-01	0.930303E 01
0.133687E 03	0.155994E 02	0.190766E 00	0.176328E-01	0.930660E 01
0.148487E 03	0.162870E 02	0.226098E 00	0.176413E-01	0.931014E 01
0.163487E 03	0.171465E 02	0.264510E 00	0.176500E-01	0.931378E 01
0.188487E 03	0.187919E 02	0.330805E 00	0.176646E-01	0.931992E 01
0.213587E 03	0.206485E 02	0.401999E 00	0.176794E-01	0.932617E 01
0.263569E 03	0.253303E 02	0.542485E 00	0.178082E-01	0.946244E 01
0.318087E 03	0.315621E 02	0.722258E 00	0.177434E-01	0.935334E 01

TABLE D-11

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.7800E 01	0.9183E 01	0.3084E 10
1	0.7625E 01	0.9794E 01	0.2975E 10
2	0.7629E 01	0.9793E 01	0.2975E 10
3	0.7629E 01	0.9793E 01	0.2975E 10

FRACTIONAL POROSITY = 0.9509E-01

SUM OF ERROR SQUARES = 0.4800E 01
VARIANCE = 0.1714E 00
STANDARD DEVIATION = 0.4140E 00

VARIANCE-COVARIANCE MATRIX

0.63627E-02	-0.39459E-01	0.28365E 07
-0.39459E-01	0.26655E 00	-0.15978E 08
0.28365E 07	-0.15978E 08	0.15636E 16

CORRELATION MATRIX

1.00000	-0.95815	0.89927
-0.95815	1.00000	-0.78265
0.89927	-0.78265	1.00000

TABLE D-12

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.13949E 02	0.13866E 02	0.59264E 00
0.14146E 02	0.14032E 02	0.80583E 00
0.14440E 02	0.14281E 02	0.11006E 01
0.14931E 02	0.14709E 02	0.14909E 01
0.15423E 02	0.15144E 02	0.18031E 01
0.15923E 02	0.15611E 02	0.19605E 01
0.16906E 02	0.16520E 02	0.22801E 01
0.17888E 02	0.17481E 02	0.22757E 01
0.19362E 02	0.18872E 02	0.25261E 01
0.20835E 02	0.20402E 02	0.20801E 01
0.22800E 02	0.22377E 02	0.18556E 01
0.25747E 02	0.25351E 02	0.15379E 01
0.28231E 02	0.27935E 02	0.10513E 01
0.31178E 02	0.31110E 02	0.21921E 00
0.35108E 02	0.35208E 02	-0.28685E 00
0.40952E 02	0.41102E 02	-0.36467E 00
0.45587E 02	0.45843E 02	-0.56166E 00
0.53537E 02	0.53883E 02	-0.64699E 00
0.63487E 02	0.63956E 02	-0.73833E 00
0.73487E 02	0.73884E 02	-0.54087E 00
0.83512E 02	0.84016E 02	-0.60357E 00
0.93487E 02	0.93857E 02	-0.39608E 00
0.10853E 03	0.10860E 03	-0.59833E-01
0.11868E 03	0.11874E 03	-0.49908E-01
0.13368E 03	0.13376E 03	-0.60561E-01
0.14848E 03	0.14825E 03	0.15566E 00
0.16348E 03	0.16333E 03	0.94173E-01
0.18848E 03	0.18807E 03	0.21719E 00
0.21358E 03	0.21340E 03	0.85128E-01
0.26356E 03	0.26248E 03	0.41113E 00
0.31808E 03	0.31904E 03	-0.29952E 00

TABLE D-13

FLOW RATE PREDICTIONS

USING THE ESTIMATED PARAMETERS K,B,AND FB

WHERE K= 0.7629E 01 MDS
B= 0.9793E 01 PSIA
FB= 0.2975E 10 1/FT

I	QQ(OBS.)	QQ(PRED.)	PERCENT ERROR
1	0.229800E-03	0.255120E-03	-0.110183E 02
2	0.323090E-03	0.358419E-03	-0.109348E 02
3	0.464950E-03	0.515241E-03	-0.108166E 02
4	0.710360E-03	0.783344E-03	-0.102742E 02
5	0.961840E-03	0.105583E-02	-0.977194E 01
6	0.121934E-02	0.132778E-02	-0.889360E 01
7	0.175037E-02	0.189188E-02	-0.808512E 01
8	0.231820E-02	0.247617E-02	-0.681446E 01
9	0.318854E-02	0.339338E-02	-0.642432E 01
10	0.416215E-02	0.435746E-02	-0.469269E 01
11	0.551059E-02	0.571914E-02	-0.378465E 01
12	0.768486E-02	0.790461E-02	-0.285959E 01
13	0.984768E-02	0.100306E-01	-0.185771E 01
14	0.125385E-01	0.125843E-01	-0.365963E 00
15	0.163457E-01	0.162674E-01	0.478489E 00
16	0.223740E-01	0.222404E-01	0.596755E 00
17	0.277845E-01	0.275327E-01	0.906139E 00
18	0.378733E-01	0.374806E-01	0.103687E 01
19	0.521510E-01	0.515359E-01	0.117932E 01
20	0.679046E-01	0.673163E-01	0.866271E 00
21	0.855086E-01	0.846845E-01	0.963738E 00
22	0.104005E 00	0.103346E 00	0.633054E 00
23	0.134069E 00	0.133937E 00	0.982081E-01
24	0.156164E 00	0.156036E 00	0.814314E-01
25	0.190766E 00	0.190581E 00	0.967654E-01
26	0.226098E 00	0.226641E 00	-0.240332E 00
27	0.264510E 00	0.264890E 00	-0.143789E 00
28	0.330805E 00	0.331889E 00	-0.327712E 00
29	0.401999E 00	0.402508E 00	-0.126726E 00
30	0.542485E 00	0.545697E 00	-0.592063E 00
31	0.722258E 00	0.719315E 00	0.407411E 00

TABLE D-14

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 1000 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-1
RUN NUMBER = 3

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.139592E 02	0.134681E 02	0.231260E-03	0.176622E-01	0.939673E 01
0.142048E 02	0.134681E 02	0.356082E-03	0.176624E-01	0.939678E 01
0.144504E 02	0.134681E 02	0.473747E-03	0.176625E-01	0.939684E 01
0.149415E 02	0.134681E 02	0.719868E-03	0.176697E-01	0.940552E 01
0.154327E 02	0.134681E 02	0.969712E-03	0.176700E-01	0.940563E 01
0.159190E 02	0.134668E 02	0.123697E-02	0.176156E-01	0.933852E 01
0.169013E 02	0.134686E 02	0.176311E-02	0.176162E-01	0.933874E 01
0.178837E 02	0.134704E 02	0.232587E-02	0.176167E-01	0.933896E 01
0.193571E 02	0.134723E 02	0.323145E-02	0.176314E-01	0.935641E 01
0.208306E 02	0.134759E 02	0.414703E-02	0.176322E-01	0.935675E 01
0.233009E 02	0.134940E 02	0.585467E-02	0.176336E-01	0.935733E 01
0.257567E 02	0.134994E 02	0.778075E-02	0.176406E-01	0.936490E 01
0.291948E 02	0.135102E 02	0.106569E-01	0.176424E-01	0.936569E 01
0.331241E 02	0.135211E 02	0.142278E-01	0.176446E-01	0.936660E 01
0.380357E 02	0.135427E 02	0.191526E-01	0.176473E-01	0.936775E 01
0.436777E 02	0.136727E 02	0.253183E-01	0.176529E-01	0.937220E 01
0.484777E 02	0.137053E 02	0.310894E-01	0.176555E-01	0.937332E 01
0.534777E 02	0.137414E 02	0.374678E-01	0.176583E-01	0.937449E 01
0.635664E 02	0.138775E 02	0.515896E-01	0.176725E-01	0.938763E 01
0.735164E 02	0.140003E 02	0.667635E-01	0.176780E-01	0.938998E 01
0.855164E 02	0.141773E 02	0.874139E-01	0.176846E-01	0.939282E 01
0.955164E 02	0.143904E 02	0.106017E 00	0.176902E-01	0.939522E 01
0.108516E 03	0.146952E 02	0.131818E 00	0.176862E-01	0.938444E 01
0.123666E 03	0.152109E 02	0.164639E 00	0.177060E-01	0.940201E 01
0.139516E 03	0.158985E 02	0.201120E 00	0.177150E-01	0.940591E 01
0.153516E 03	0.165862E 02	0.234427E 00	0.177231E-01	0.940938E 01
0.178516E 03	0.181087E 02	0.300026E 00	0.177376E-01	0.941567E 01
0.213516E 03	0.207364E 02	0.395313E 00	0.177723E-01	0.944212E 01
0.264816E 03	0.255007E 02	0.543868E 00	0.178033E-01	0.945567E 01
0.313716E 03	0.308052E 02	0.694392E 00	0.178334E-01	0.946892E 01

TABLE D-15

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.7630E 01	0.9793E 01	0.2975E 10
1	0.7591E 01	0.9912E 01	0.3065E 10
2	0.7592E 01	0.9908E 01	0.3066E 10
3	0.7592E 01	0.9908E 01	0.3066E 10

FRACTIONAL POROSITY = 0.9267E-01

SUM OF ERROR SQUARES = 0.2882E 01
VARIANCE = 0.1067E 00
STANDARD DEVIATION = 0.3267E 00

VARIANCE-COVARIANCE MATRIX

0.43856E-02	-0.27008E-01	0.20207E 07
-0.27008E-01	0.18028E 00	-0.11403E 08
0.20207E 07	-0.11403E 08	0.11339E 16

CORRELATION MATRIX

1.00000	-0.96052	0.90612
-0.96052	1.00000	-0.79752
0.90612	-0.79752	1.00000

TABLE D-16

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.13959E 02	0.13879E 02	0.56823E 00
0.14204E 02	0.14111E 02	0.65999E 00
0.14450E 02	0.14315E 02	0.93081E 00
0.14941E 02	0.14752E 02	0.12650E 01
0.15432E 02	0.15186E 02	0.15931E 01
0.15919E 02	0.15635E 02	0.17799E 01
0.16901E 02	0.16513E 02	0.22958E 01
0.17883E 02	0.17443E 02	0.24622E 01
0.19357E 02	0.18915E 02	0.22794E 01
0.20830E 02	0.20300E 02	0.25456E 01
0.23300E 02	0.22798E 02	0.21548E 01
0.25756E 02	0.25460E 02	0.11518E 01
0.29194E 02	0.29072E 02	0.41750E 00
0.33124E 02	0.33142E 02	-0.54495E-01
0.38035E 02	0.38229E 02	-0.50838E 00
0.43677E 02	0.44016E 02	-0.77510E 00
0.48477E 02	0.48923E 02	-0.91988E 00
0.53477E 02	0.53946E 02	-0.87579E 00
0.63566E 02	0.64077E 02	-0.80460E 00
0.73516E 02	0.73759E 02	-0.33013E 00
0.85516E 02	0.85728E 02	-0.24784E 00
0.95516E 02	0.95649E 02	-0.13907E 00
0.10851E 03	0.10834E 03	0.15640E 00
0.12366E 03	0.12357E 03	0.71416E-01
0.13951E 03	0.13933E 03	0.13098E 00
0.15351E 03	0.15298E 03	0.34945E 00
0.17851E 03	0.17849E 03	0.95732E-02
0.21351E 03	0.21346E 03	0.25827E-01
0.26481E 03	0.26456E 03	0.95764E-01
0.31371E 03	0.31406E 03	-0.10994E 00

TABLE D-17

A SEMI-GRAPHICAL METHOD OF PARAMETER ESTIMATION
I.E. A SIMULTANEOUS LINEAR LEAST SQUARES FIT TECHNIQUE

NV	B(NV)	KV(NV)	KI(NI)	FB(NI)	NRE(NV)
4	0.1053E 01	0.1107E 02	0.1018E 02	0.4340E 10	0.1452E-02
5	0.3669E 01	0.9420E 01	0.9154E 01	0.3886E 10	0.1741E-02
6	0.4000E 01	0.9246E 01	0.9030E 01	0.3822E 10	0.2181E-02
7	0.5640E 01	0.8475E 01	0.8536E 01	0.3572E 10	0.2874E-02
8	0.5777E 01	0.8416E 01	0.8503E 01	0.3557E 10	0.3737E-02
9	0.4899E 01	0.8803E 01	0.8734E 01	0.3669E 10	0.5303E-02
10	0.5135E 01	0.8696E 01	0.8666E 01	0.3634E 10	0.6676E-02
11	0.5079E 01	0.8721E 01	0.8671E 01	0.3633E 10	0.9271E-02
12	0.4620E 01	0.8929E 01	0.8758E 01	0.3660E 10	0.1228E-01
13	0.4436E 01	0.9014E 01	0.8764E 01	0.3646E 10	0.1655E-01
14	0.4522E 01	0.8975E 01	0.8701E 01	0.3601E 10	0.2150E-01
15	0.4757E 01	0.8869E 01	0.8603E 01	0.3542E 10	0.2794E-01
16	0.5097E 01	0.8721E 01	0.8487E 01	0.3475E 10	0.3546E-01
17	0.5434E 01	0.8581E 01	0.8372E 01	0.3407E 10	0.4192E-01
18	0.5789E 01	0.8439E 01	0.8256E 01	0.3338E 10	0.4861E-01
19	0.6263E 01	0.8258E 01	0.8129E 01	0.3267E 10	0.6362E-01
20	0.6828E 01	0.8055E 01	0.8009E 01	0.3208E 10	0.7863E-01
21	0.7416E 01	0.7856E 01	0.7896E 01	0.3156E 10	0.9822E-01
22	0.7978E 01	0.7675E 01	0.7793E 01	0.3108E 10	0.1143E 00
23	0.8573E 01	0.7495E 01	0.7709E 01	0.3077E 10	0.1369E 00
24	0.9175E 01	0.7321E 01	0.7622E 01	0.3044E 10	0.1642E 00
25	0.9795E 01	0.7152E 01	0.7541E 01	0.3016E 10	0.1933E 00
26	0.1042E 02	0.6990E 01	0.7512E 01	0.3029E 10	0.2206E 00

DATA DISCRIMINATION METHODS

METHOD 1. FINDING THE BEST FIT FOR THE KLINKENBERG PLOT
I.E. THE MAXIMUM CORRELATION COEFFICIENT
NV= 15 R(NV)= 0.990 (MAXIMUM)
NI= 15 R(NI)= 0.994

METHOD 2. MATCHING THE VALUES OF K FROM THE KLINKENBERG AND MODIFIED VISCO-INERTIAL PLOTS
I.E. THE MINIMUM RELATIVE ERROR BETWEEN KV AND KI
NV= 10
NI= 20
ERRK(NV)= 0.3466E 00 (MIN. PERCENT)

METHOD 3. THE REYNOLDS NUMBER CRITERION, I.E. THE POINTS ARE IN VISCOUS-REGION IF NRE IS LESS THAN 0.01
NV= 11
NI= 19
NRE(NV)= 0.9271E-02

TABLE D-18

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 1200 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-1
RUN NUMBER = 4

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.138915E 02	0.134004E 02	0.227640E-03	0.176594E-01	0.939327E 01
0.143827E 02	0.134004E 02	0.464610E-03	0.176596E-01	0.939338E 01
0.148739E 02	0.134004E 02	0.727710E-03	0.176599E-01	0.939349E 01
0.153747E 02	0.134137E 02	0.977410E-03	0.176545E-01	0.938661E 01
0.163570E 02	0.134155E 02	0.150503E-02	0.176550E-01	0.938683E 01
0.173393E 02	0.134173E 02	0.203980E-02	0.176556E-01	0.938706E 01
0.188128E 02	0.134191E 02	0.291027E-02	0.176564E-01	0.938741E 01
0.202863E 02	0.134227E 02	0.384336E-02	0.176572E-01	0.938775E 01
0.222509E 02	0.134281E 02	0.521284E-02	0.176582E-01	0.938821E 01
0.251979E 02	0.134326E 02	0.739429E-02	0.176598E-01	0.938889E 01
0.281642E 02	0.134583E 02	0.969760E-02	0.176615E-01	0.938959E 01
0.330758E 02	0.134763E 02	0.141670E-01	0.176641E-01	0.939073E 01
0.389697E 02	0.134998E 02	0.203327E-01	0.176673E-01	0.939211E 01
0.487512E 02	0.137498E 02	0.316181E-01	0.176644E-01	0.938416E 01
0.585512E 02	0.138257E 02	0.447906E-01	0.176698E-01	0.938645E 01
0.704518E 02	0.140447E 02	0.623948E-01	0.176820E-01	0.939615E 01
0.836518E 02	0.142412E 02	0.845436E-01	0.176892E-01	0.939928E 01
0.986518E 02	0.145408E 02	0.112728E 00	0.176976E-01	0.940287E 01
0.113651E 03	0.149435E 02	0.143068E 00	0.177059E-01	0.940649E 01
0.133651E 03	0.157245E 02	0.187895E 00	0.177201E-01	0.941483E 01
0.153651E 03	0.167068E 02	0.236243E 00	0.177315E-01	0.941980E 01
0.183751E 03	0.185511E 02	0.313744E 00	0.177490E-01	0.942739E 01
0.213751E 03	0.208472E 02	0.397518E 00	0.177668E-01	0.943512E 01
0.263451E 03	0.256852E 02	0.541237E 00	0.178026E-01	0.945538E 01
0.313851E 03	0.310634E 02	0.702137E 00	0.178336E-01	0.946901E 01

TABLE D-19

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.7592E 01	0.9908E 01	0.3066E 10
1	0.7508E 01	0.1053E 02	0.2952E 10
2	0.7510E 01	0.1052E 02	0.2953E 10
3	0.7510E 01	0.1052E 02	0.2953E 10

FRACTIONAL POROSITY = 0.9100E-01

SUM OF ERROR SQUARES = 0.5460E 01
VARIANCE = 0.2482E 00
STANDARD DEVIATION = 0.4982E 00

VARIANCE-COVARIANCE MATRIX

0.10963E-01	-0.69974E-01	0.50502E 07
-0.69974E-01	0.48708E 00	-0.29368E 08
0.50502E 07	-0.29368E 08	0.28145E 16

CORRELATION MATRIX

1.00000	-0.95752	0.90912
-0.95752	1.00000	-0.79318
0.90912	-0.79318	1.00000

TABLE D-20

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.13891E 02	0.13788E 02	0.74412E 00
0.14382E 02	0.14195E 02	0.12990E 01
0.14873E 02	0.14674E 02	0.13403E 01
0.15374E 02	0.15109E 02	0.17230E 01
0.16357E 02	0.16005E 02	0.21482E 01
0.17339E 02	0.16874E 02	0.26789E 01
0.18812E 02	0.18274E 02	0.28596E 01
0.20286E 02	0.19727E 02	0.27550E 01
0.22250E 02	0.21785E 02	0.20902E 01
0.25197E 02	0.24799E 02	0.15802E 01
0.28164E 02	0.27716E 02	0.15910E 01
0.33075E 02	0.32925E 02	0.45396E 00
0.38969E 02	0.39290E 02	-0.82355E 00
0.48751E 02	0.49285E 02	-0.10966E 01
0.58551E 02	0.59284E 02	-0.12516E 01
0.70451E 02	0.71024E 02	-0.81247E 00
0.83651E 02	0.84082E 02	-0.51488E 00
0.98651E 02	0.99004E 02	-0.35766E 00
0.11365E 03	0.11364E 03	0.94786E-02
0.13365E 03	0.13358E 03	0.46306E-01
0.15365E 03	0.15350E 03	0.95156E-01
0.18375E 03	0.18323E 03	0.28394E 00
0.21375E 03	0.21343E 03	0.14659E 00
0.26345E 03	0.26250E 03	0.35948E 00
0.31385E 03	0.31480E 03	-0.30386E 00

TABLE D-21

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 800 PSIG
TEST-METHOD USED = CASE-II
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-1
RUN NUMBER = 5

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.313035E 03	0.307857E 02	0.702130E 00	0.177060E-01	0.930898E 01
0.313535E 03	0.536857E 02	0.688457E 00	0.177191E-01	0.931446E 01
0.313535E 03	0.735358E 02	0.675655E 00	0.177302E-01	0.931917E 01
0.313535E 03	0.935358E 02	0.656990E 00	0.177415E-01	0.932394E 01
0.314535E 03	0.123735E 03	0.623632E 00	0.177591E-01	0.933145E 01
0.314535E 03	0.153335E 03	0.576241E 00	0.177759E-01	0.933863E 01
0.315035E 03	0.183535E 03	0.518047E 00	0.177934E-01	0.934615E 01
0.315035E 03	0.214335E 03	0.443413E 00	0.178111E-01	0.935376E 01
0.315035E 03	0.233185E 03	0.386764E 00	0.178302E-01	0.936895E 01
0.315535E 03	0.253535E 03	0.312742E 00	0.178422E-01	0.937419E 01
0.313535E 03	0.263535E 03	0.268175E 00	0.178241E-01	0.934711E 01
0.313535E 03	0.273035E 03	0.228414E 00	0.178296E-01	0.934950E 01
0.313535E 03	0.278135E 03	0.205572E 00	0.178326E-01	0.935079E 01
0.313535E 03	0.283135E 03	0.181874E 00	0.178355E-01	0.935205E 01
0.313535E 03	0.289535E 03	0.149096E 00	0.178513E-01	0.936905E 01
0.313535E 03	0.293535E 03	0.127330E 00	0.178536E-01	0.937007E 01
0.313535E 03	0.297535E 03	0.104041E 00	0.178559E-01	0.937109E 01
0.313535E 03	0.301535E 03	0.815150E-01	0.178583E-01	0.937211E 01
0.313535E 03	0.303535E 03	0.688587E-01	0.178595E-01	0.937262E 01

TABLE D-22

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.7630E 01	0.9793E 01	0.2975E 10
1	0.7758E 01	0.9978E 01	0.3138E 10
2	0.7760E 01	0.9963E 01	0.3137E 10
3	0.7760E 01	0.9963E 01	0.3137E 10

FRACTIONAL POROSITY = 0.9106E-01

SUM OF ERROR SQUARES = 0.2287E 01
VARIANCE = 0.1429E 00
STANDARD DEVIATION = 0.3780E 00

VARIANCE-COVARIANCE MATRIX

0.25090E-02	-0.56066E-01	-0.50278E 06
-0.56066E-01	0.70321E 01	0.24129E 09
-0.50278E 06	0.24129E 09	0.93211E 16

CORRELATION MATRIX

1.00000	-0.42209	-0.10396
-0.42209	1.00000	0.94247
-0.10396	0.94247	1.00000

TABLE D-23

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.31303E 03	0.31352E 03	-0.15758E 00
0.31353E 03	0.31292E 03	0.19620E 00
0.31353E 03	0.31349E 03	0.12945E-01
0.31353E 03	0.31356E 03	-0.88573E-02
0.31453E 03	0.31463E 03	-0.31455E-01
0.31453E 03	0.31446E 03	0.21131E-01
0.31503E 03	0.31501E 03	0.66259E-02
0.31503E 03	0.31551E 03	-0.15076E 00
0.31503E 03	0.31541E 03	-0.12035E 00
0.31553E 03	0.31451E 03	0.32247E 00
0.31353E 03	0.31327E 03	0.83414E-01
0.31353E 03	0.31348E 03	0.15690E-01
0.31353E 03	0.31359E 03	-0.20031E-01
0.31353E 03	0.31367E 03	-0.44637E-01
0.31353E 03	0.31370E 03	-0.53222E-01
0.31353E 03	0.31367E 03	-0.45007E-01
0.31353E 03	0.31357E 03	-0.10920E-01
0.31353E 03	0.31378E 03	-0.78528E-01
0.31353E 03	0.31373E 03	-0.63539E-01

TABLE D-24

FLOW RATE PREDICTIONS

USING THE ESTIMATED PARAMETERS K,B,AND FB

WHERE K= 0.7629E 01 MDS
B= 0.9793E 01 PSIA
FB= 0.2975E 10 1/FT

I	QO(OBS.)	QO(PRED.)	PERCENT ERROR
1	0.702130E 00	0.705442E 00	-0.471791E 00
2	0.688457E 00	0.695011E 00	-0.952071E 00
3	0.675655E 00	0.680198E 00	-0.672482E 00
4	0.656990E 00	0.661004E 00	-0.611097E 00
5	0.623632E 00	0.626857E 00	-0.517146E 00
6	0.576241E 00	0.579262E 00	-0.524322E 00
7	0.518047E 00	0.520029E 00	-0.382700E 00
8	0.443413E 00	0.442420E 00	0.223840E 00
9	0.386764E 00	0.385376E 00	0.358817E 00
10	0.312742E 00	0.316781E 00	-0.129153E 01
11	0.268175E 00	0.268544E 00	-0.137712E 00
12	0.228414E 00	0.227573E 00	0.368043E 00
13	0.205572E 00	0.204080E 00	0.725762E 00
14	0.181874E 00	0.179910E 00	0.107973E 01
15	0.149096E 00	0.146940E 00	0.144576E 01
16	0.127330E 00	0.125331E 00	0.156932E 01
17	0.104041E 00	0.102732E 00	0.125810E 01
18	0.815150E-01	0.790382E-01	0.303847E 01
19	0.688587E-01	0.667430E-01	0.307247E 01
20	0.546039E-01	0.571507E-01	-0.466427E 01
21	0.415392E-01	0.443930E-01	-0.687021E 01
22	0.273420E-01	0.311755E-01	-0.140205E 02
23	0.189611E-01	0.244253E-01	-0.288180E 02
24	0.138405E-01	0.175765E-01	-0.269933E 02
25	0.802349E-02	0.141140E-01	-0.759096E 02
26	0.413150E-02	0.106256E-01	-0.157187E 03
27	0.225968E-02	0.568865E-02	-0.151746E 03
28	0.104457E-02	0.712152E-02	-0.581766E 03
29	0.447190E-03	0.427292E-02	-0.855504E 03

TABLE D-25

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 1000 PSIG
TEST-METHOD USED = CASE-II
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-1
RUN NUMBER = 6

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.312997E 03	0.306471E 02	0.689950E 00	0.178357E-01	0.947219E 01
0.313597E 03	0.105847E 03	0.641191E 00	0.178782E-01	0.949100E 01
0.315197E 03	0.163797E 03	0.553459E 00	0.179120E-01	0.950602E 01
0.313497E 03	0.193497E 03	0.484147E 00	0.179335E-01	0.952004E 01
0.313497E 03	0.223097E 03	0.406223E 00	0.179505E-01	0.952766E 01
0.313497E 03	0.243497E 03	0.341774E 00	0.179622E-01	0.953295E 01
0.313497E 03	0.263497E 03	0.268261E 00	0.179738E-01	0.953817E 01
0.313497E 03	0.273497E 03	0.226022E 00	0.179797E-01	0.954079E 01
0.313997E 03	0.284297E 03	0.176880E 00	0.179862E-01	0.954375E 01
0.313997E 03	0.293697E 03	0.128684E 00	0.179917E-01	0.954623E 01
0.313997E 03	0.299497E 03	0.965904E-01	0.179951E-01	0.954776E 01

TABLE D-26

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.7592E 01	0.9908E 01	0.3066E 10
1	0.7979E 01	0.2803E 01	0.3005E 10
2	0.7998E 01	0.2832E 01	0.3005E 10
3	0.7999E 01	0.2831E 01	0.3005E 10
4	0.7999E 01	0.2831E 01	0.3005E 10

FRACTIONAL POROSITY = 0.2172E 00

SUM OF ERROR SQUARES = 0.2489E 01
VARIANCE = 0.3111E 00
STANDARD DEVIATION = 0.5578E 00

VARIANCE-COVARIANCE MATRIX

0.82766E-02	-0.11991E 00	0.88634E 06
-0.11991E 00	0.19073E 02	0.69107E 09
0.88634E 06	0.69107E 09	0.28961E 17

CORRELATION MATRIX

1.00000	-0.30179	0.05724
-0.30179	1.00000	0.92982
0.05724	0.92982	1.00000

TABLE D-27

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.31299E 03	0.31267E 03	0.10235E 00
0.31359E 03	0.31474E 03	-0.36703E 00
0.31519E 03	0.31459E 03	0.19034E 00
0.31349E 03	0.31300E 03	0.15598E 00
0.31349E 03	0.31318E 03	0.97988E-01
0.31349E 03	0.31338E 03	0.35667E-01
0.31349E 03	0.31372E 03	-0.73612E-01
0.31349E 03	0.31380E 03	-0.99175E-01
0.31399E 03	0.31410E 03	-0.35144E-01
0.31399E 03	0.31423E 03	-0.76450E-01
0.31399E 03	0.31436E 03	-0.11762E 00

TABLE D-28

FLOW RATE PREDICTIONS

USING THE ESTIMATED PARAMETERS K,B,AND FB

WHERE K= 0.7592E 01 MDS
B= 0.9908E 01 PSIA
FB= 0.3066E 10 1/FT

I	QO(OBS.)	QO(PRED.)	PERCENT ERROR
1	0.689950E 00	0.690992E 00	-0.151043E 00
2	0.641191E 00	0.633879E 00	0.114025E 01
3	0.553459E 00	0.550391E 00	0.554240E 00
4	0.484147E 00	0.480415E 00	0.770784E 00
5	0.406223E 00	0.401827E 00	0.108216E 01
6	0.341774E 00	0.336844E 00	0.144240E 01
7	0.268261E 00	0.262289E 00	0.222609E 01
8	0.226022E 00	0.220036E 00	0.264816E 01
9	0.176880E 00	0.172502E 00	0.247464E 01
10	0.128684E 00	0.124311E 00	0.339771E 01
11	0.965904E-01	0.919930E-01	0.475971E 01
12	0.703834E-01	0.683181E-01	0.293425E 01
13	0.445137E-01	0.441001E-01	0.929044E 00
14	0.289869E-01	0.292069E-01	-0.759236E 00
15	0.224200E-01	0.239123E-01	-0.665617E 01
16	0.142409E-01	0.158540E-01	-0.113278E 02
17	0.106897E-01	0.131311E-01	-0.228390E 02
18	0.775766E-02	0.103981E-01	-0.340378E 02
19	0.255667E-02	0.626720E-02	-0.145131E 03
20	0.769173E-03	0.488226E-02	-0.534742E 03

TABLE D-29

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 400 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-2
RUN NUMBER = 1

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.186421E 02	0.136421E 02	0.317505E-03	0.175376E-01	0.924137E 01
0.216421E 02	0.136421E 02	0.544934E-03	0.175377E-01	0.924014E 01
0.236131E 02	0.136131E 02	0.700758E-03	0.176452E-01	0.937144E 01
0.296131E 02	0.136131E 02	0.126621E-02	0.176509E-01	0.937594E 01
0.336131E 02	0.136131E 02	0.170294E-02	0.176139E-01	0.932859E 01
0.356131E 02	0.136131E 02	0.194235E-02	0.176232E-01	0.933917E 01
0.416131E 02	0.136131E 02	0.271913E-02	0.176178E-01	0.932980E 01
0.536131E 02	0.136131E 02	0.464309E-02	0.176305E-01	0.934018E 01
0.635744E 02	0.135871E 02	0.660774E-02	0.175480E-01	0.923413E 01
0.736131E 02	0.136131E 02	0.875923E-02	0.176326E-01	0.933401E 01
0.886131E 02	0.136402E 02	0.126065E-01	0.176493E-01	0.934793E 01
0.113651E 03	0.136528E 02	0.205451E-01	0.176341E-01	0.931802E 01
0.133651E 03	0.136536E 02	0.280889E-01	0.176507E-01	0.932967E 01
0.153651E 03	0.136536E 02	0.367375E-01	0.176616E-01	0.933429E 01
0.173651E 03	0.136545E 02	0.463541E-01	0.176789E-01	0.934679E 01
0.193651E 03	0.136554E 02	0.568132E-01	0.176899E-01	0.935145E 01
0.213651E 03	0.136554E 02	0.681754E-01	0.177009E-01	0.935613E 01
0.243613E 03	0.139078E 02	0.879439E-01	0.176922E-01	0.933159E 01
0.313642E 03	0.141333E 02	0.141621E 00	0.177312E-01	0.934814E 01
0.463642E 03	0.150173E 02	0.295529E 00	0.178161E-01	0.938466E 01

TABLE D-30

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.9000E 00	0.8390E 01	0.2547E 10
1	0.7621E 00	0.6580E 01	0.1511E 11
2	0.7765E 00	0.6347E 01	0.1641E 11
3	0.7767E 00	0.6349E 01	0.1640E 11
4	0.7767E 00	0.6349E 01	0.1640E 11

FRACTIONAL POROSITY = 0.4575E-01

SUM OF ERROR SQUARES = 0.5819E 01
VARIANCE = 0.3423E 00
STANDARD DEVIATION = 0.5850E 00

VARIANCE-COVARIANCE MATRIX

0.25713E-04	-0.21929E-02	0.52499E 07
-0.21929E-02	0.22349E 00	-0.39770E 09
0.52499E 07	-0.39770E 09	0.13708E 19

CORRELATION MATRIX

1.00000	-0.91476	0.88424
-0.91476	1.00000	-0.71848
0.88424	-0.71848	1.00000

TABLE D-31

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.18642E 02	0.18389E 02	0.13564E 01
0.21642E 02	0.21317E 02	0.14991E 01
0.23613E 02	0.23255E 02	0.15146E 01
0.29613E 02	0.29232E 02	0.12868E 01
0.33613E 02	0.33148E 02	0.13818E 01
0.35613E 02	0.35201E 02	0.11548E 01
0.41613E 02	0.41166E 02	0.10725E 01
0.53613E 02	0.53546E 02	0.12464E 00
0.63574E 02	0.63564E 02	0.15804E-01
0.73613E 02	0.73784E 02	-0.23317E 00
0.88613E 02	0.89004E 02	-0.44135E 00
0.11365E 03	0.11428E 03	-0.56082E 00
0.13365E 03	0.13441E 03	-0.57223E 00
0.15365E 03	0.15447E 03	-0.53818E 00
0.17365E 03	0.17436E 03	-0.41114E 00
0.19365E 03	0.19380E 03	-0.81600E-01
0.21365E 03	0.21311E 03	0.25343E 00
0.24361E 03	0.24305E 03	0.22764E 00
0.31364E 03	0.31237E 03	0.40261E 00
0.46364E 03	0.46409E 03	-0.96770E-01

TABLE D-32

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 400 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-2
RUN NUMBER = 2

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.155004E 02	0.135357E 02	0.151305E-03	0.177636E-01	0.952110E 01
0.164827E 02	0.135357E 02	0.232638E-03	0.177641E-01	0.952134E 01
0.174650E 02	0.135357E 02	0.314895E-03	0.177646E-01	0.952157E 01
0.184280E 02	0.135164E 02	0.403225E-03	0.177652E-01	0.952179E 01
0.194103E 02	0.135164E 02	0.493603E-03	0.177657E-01	0.952202E 01
0.208838E 02	0.135164E 02	0.635922E-03	0.177609E-01	0.951550E 01
0.223573E 02	0.135164E 02	0.784365E-03	0.177617E-01	0.951584E 01
0.243219E 02	0.135164E 02	0.991529E-03	0.177628E-01	0.951631E 01
0.262865E 02	0.135200E 02	0.121506E-02	0.177639E-01	0.951678E 01
0.282512E 02	0.135218E 02	0.145005E-02	0.177649E-01	0.951724E 01
0.307070E 02	0.135227E 02	0.175597E-02	0.177662E-01	0.951782E 01
0.341451E 02	0.135236E 02	0.223280E-02	0.177681E-01	0.951864E 01
0.380744E 02	0.135254E 02	0.280867E-02	0.177702E-01	0.951957E 01
0.425164E 02	0.135272E 02	0.353289E-02	0.177727E-01	0.952062E 01
0.485164E 02	0.135309E 02	0.459784E-02	0.177759E-01	0.952205E 01
0.542971E 02	0.135133E 02	0.574499E-02	0.177736E-01	0.951670E 01
0.622971E 02	0.135187E 02	0.748938E-02	0.177780E-01	0.951860E 01
0.722971E 02	0.135278E 02	0.998815E-02	0.177834E-01	0.952099E 01
0.872971E 02	0.135422E 02	0.144502E-01	0.177916E-01	0.952457E 01
0.102297E 03	0.135621E 02	0.196210E-01	0.177997E-01	0.952816E 01
0.123497E 03	0.136018E 02	0.281035E-01	0.178113E-01	0.953326E 01
0.143497E 03	0.137571E 02	0.377107E-01	0.178223E-01	0.953812E 01
0.163497E 03	0.138257E 02	0.482363E-01	0.178333E-01	0.954298E 01
0.183497E 03	0.139124E 02	0.603414E-01	0.178443E-01	0.954786E 01
0.203497E 03	0.140171E 02	0.737215E-01	0.178554E-01	0.955276E 01
0.223497E 03	0.141543E 02	0.885470E-01	0.178664E-01	0.955769E 01
0.253497E 03	0.144057E 02	0.112489E 00	0.178831E-01	0.956515E 01
0.293497E 03	0.148969E 02	0.149721E 00	0.179055E-01	0.957520E 01
0.333497E 03	0.156336E 02	0.191523E 00	0.179282E-01	0.958542E 01
0.373497E 03	0.165914E 02	0.238514E 00	0.179512E-01	0.959579E 01
0.413497E 03	0.177702E 02	0.290757E 00	0.179744E-01	0.960634E 01
0.473497E 03	0.199313E 02	0.376675E 00	0.180097E-01	0.962249E 01
0.562497E 03	0.242043E 02	0.518118E 00	0.180635E-01	0.964727E 01
0.659497E 03	0.303193E 02	0.705058E 00	0.181242E-01	0.967552E 01
0.796497E 03	0.411493E 02	0.100775E 01	0.182138E-01	0.971776E 01

TABLE D-33

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.9000E 00	0.8390E 01	0.2547E 10
1	0.9081E 00	0.8391E 01	0.2547E 10
2	0.9081E 00	0.8391E 01	0.2547E 10
3	0.9081E 00	0.8391E 01	0.2547E 10

FRACTIONAL POROSITY = 0.1157E 00

SUM OF ERROR SQUARES = 0.8219E 01
VARIANCE = 0.2568E 00
STANDARD DEVIATION = 0.5068E 00

VARIANCE-COVARIANCE MATRIX

0.47758E-05	-0.48218E-03	0.25318E 06
-0.48218E-03	0.65222E-01	-0.22040E 08
0.25318E 06	-0.22040E 08	0.16868E 17

CORRELATION MATRIX

1.00000	-0.86395	0.89203
-0.86395	1.00000	-0.66447
0.89203	-0.66447	1.00000

TABLE D-34

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.15500E 02	0.15516E 02	-0.10130E 00
0.16482E 02	0.16520E 02	-0.22754E 00
0.17465E 02	0.17482E 02	-0.10060E 00
0.18428E 02	0.18479E 02	-0.27732E 00
0.19410E 02	0.19469E 02	-0.30536E 00
0.20883E 02	0.20957E 02	-0.35303E 00
0.22357E 02	0.22433E 02	-0.33998E 00
0.24321E 02	0.24375E 02	-0.22064E 00
0.26286E 02	0.26363E 02	-0.29354E 00
0.28251E 02	0.28332E 02	-0.28660E 00
0.30707E 02	0.30733E 02	-0.86165E-01
0.34145E 02	0.34237E 02	-0.26935E 00
0.38074E 02	0.38091E 02	-0.44164E-01
0.42516E 02	0.42543E 02	-0.64331E-01
0.48516E 02	0.48446E 02	0.14355E 00
0.54297E 02	0.54166E 02	0.24085E 00
0.62297E 02	0.62004E 02	0.47028E 00
0.72297E 02	0.71931E 02	0.50516E 00
0.87297E 02	0.87170E 02	0.14505E 00
0.10229E 03	0.10223E 03	0.61708E-01
0.12349E 03	0.12327E 03	0.17927E 00
0.14349E 03	0.14371E 03	-0.14971E 00
0.16349E 03	0.16331E 03	0.11391E 00
0.18349E 03	0.18345E 03	0.23782E-01
0.20349E 03	0.20353E 03	-0.20770E-01
0.22349E 03	0.22382E 03	-0.14716E 00
0.25349E 03	0.25334E 03	0.60072E-01
0.29349E 03	0.29377E 03	-0.95328E-01
0.33349E 03	0.33373E 03	-0.71010E-01
0.37349E 03	0.37394E 03	-0.11873E 00
0.41349E 03	0.41444E 03	-0.22840E 00
0.47349E 03	0.47421E 03	-0.15113E 00
0.56249E 03	0.56023E 03	0.40221E 00
0.65949E 03	0.65899E 03	0.75889E-01
0.79649E 03	0.79723E 03	-0.92599E-01

TABLE D-35

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 600 PSIG
TEST-METHOD USED = CASE-1
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-2
RUN NUMBER = 3

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.155197E 02	0.135551E 02	0.120442E-03	0.176408E-01	0.936956E 01
0.165020E 02	0.135551E 02	0.184392E-03	0.176413E-01	0.936978E 01
0.174844E 02	0.135551E 02	0.249754E-03	0.176418E-01	0.937001E 01
0.184667E 02	0.135551E 02	0.318296E-03	0.176424E-01	0.937024E 01
0.194539E 02	0.135599E 02	0.390155E-03	0.176429E-01	0.937046E 01
0.209273E 02	0.135599E 02	0.501276E-03	0.176437E-01	0.937081E 01
0.224008E 02	0.135599E 02	0.617751E-03	0.176445E-01	0.937114E 01
0.243654E 02	0.135599E 02	0.779933E-03	0.176455E-01	0.937160E 01
0.263300E 02	0.135599E 02	0.951419E-03	0.176466E-01	0.937205E 01
0.282947E 02	0.135617E 02	0.113689E-02	0.176477E-01	0.937251E 01
0.307505E 02	0.135635E 02	0.138063E-02	0.176490E-01	0.937308E 01
0.331940E 02	0.135653E 02	0.163950E-02	0.176503E-01	0.937364E 01
0.371356E 02	0.135662E 02	0.208563E-02	0.176525E-01	0.937456E 01
0.420471E 02	0.135671E 02	0.268718E-02	0.176551E-01	0.937570E 01
0.455599E 02	0.135689E 02	0.316363E-02	0.176596E-01	0.937963E 01
0.515599E 02	0.135707E 02	0.405493E-02	0.176628E-01	0.938103E 01
0.595599E 02	0.135744E 02	0.537029E-02	0.176672E-01	0.938288E 01
0.695599E 02	0.135798E 02	0.726695E-02	0.176726E-01	0.938521E 01
0.795599E 02	0.135870E 02	0.938680E-02	0.176781E-01	0.938755E 01
0.895648E 02	0.136009E 02	0.118226E-01	0.176835E-01	0.938989E 01
0.103564E 03	0.136207E 02	0.156501E-01	0.176911E-01	0.939316E 01
0.123564E 03	0.136370E 02	0.218220E-01	0.177021E-01	0.939787E 01
0.143764E 03	0.138609E 02	0.293605E-01	0.177132E-01	0.940267E 01
0.163564E 03	0.138970E 02	0.374691E-01	0.177297E-01	0.941442E 01
0.183564E 03	0.139602E 02	0.469996E-01	0.177407E-01	0.941918E 01
0.203564E 03	0.140342E 02	0.574167E-01	0.177518E-01	0.942397E 01
0.223464E 03	0.141209E 02	0.682944E-01	0.177627E-01	0.942875E 01
0.253364E 03	0.142892E 02	0.873207E-01	0.177793E-01	0.943597E 01
0.293574E 03	0.145936E 02	0.117173E 00	0.177822E-01	0.942123E 01
0.333674E 03	0.150233E 02	0.149999E 00	0.178048E-01	0.943106E 01
0.373574E 03	0.156127E 02	0.186949E 00	0.178274E-01	0.944098E 01
0.413574E 03	0.163495E 02	0.228160E 00	0.178504E-01	0.945106E 01
0.473574E 03	0.178229E 02	0.296003E 00	0.178853E-01	0.946645E 01
0.561574E 03	0.207945E 02	0.410256E 00	0.179715E-01	0.953267E 01
0.663574E 03	0.255587E 02	0.564846E 00	0.180343E-01	0.956102E 01

TABLE D-36

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.9081E 00	0.9391E 01	0.2547E 10
1	0.6501E 00	0.8640E 01	0.2029E 10
2	0.6937E 00	0.8373E 01	0.1856E 10
3	0.6959E 00	0.8376E 01	0.1857E 10
4	0.6960E 00	0.8376E 01	0.1857E 10

FRACTIONAL POROSITY = 0.1400E 00

SUM OF ERROR SQUARES = 0.2349E 01
VARIANCE = 0.7341E-01
STANDARD DEVIATION = 0.2709E 00

VARIANCE-COVARIANCE MATRIX

0.11619E-05	-0.13711E-03	0.18254E 06
-0.13711E-03	0.21106E-01	-0.18706E 08
0.18254E 06	-0.18706E 08	0.35534E 17

CORRELATION MATRIX

1.00000	-0.87556	0.89837
-0.87556	1.00000	-0.68306
0.89837	-0.68306	1.00000

TABLE D-37

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.15519E 02	0.15599E 02	-0.51259E 00
0.16502E 02	0.16613E 02	-0.67460E 00
0.17484E 02	0.17599E 02	-0.66101E 00
0.18466E 02	0.18599E 02	-0.72149E 00
0.19453E 02	0.19617E 02	-0.84004E 00
0.20927E 02	0.21107E 02	-0.86132E 00
0.22400E 02	0.22590E 02	-0.84778E 00
0.24365E 02	0.24536E 02	-0.70318E 00
0.26330E 02	0.26475E 02	-0.55060E 00
0.28294E 02	0.28471E 02	-0.62580E 00
0.30750E 02	0.30934E 02	-0.59778E 00
0.33194E 02	0.33392E 02	-0.59894E 00
0.37135E 02	0.37310E 02	-0.47209E 00
0.42047E 02	0.42100E 02	-0.12735E 00
0.45559E 02	0.45617E 02	-0.12695E 00
0.51559E 02	0.51638E 02	-0.15161E 00
0.59559E 02	0.59542E 02	0.29577E-01
0.69559E 02	0.69561E 02	-0.24129E-02
0.79559E 02	0.79405E 02	0.19460E 00
0.89564E 02	0.89532E 02	0.35674E-01
0.10356E 03	0.10361E 03	-0.44907E-01
0.12356E 03	0.12315E 03	0.33171E 00
0.14376E 03	0.14375E 03	0.40119E-02
0.16356E 03	0.16320E 03	0.22154E 00
0.18356E 03	0.18356E 03	0.21446E-02
0.20356E 03	0.20360E 03	-0.17929E-01
0.22346E 03	0.22267E 03	0.35254E 00
0.25336E 03	0.25281E 03	0.21645E 00
0.29357E 03	0.29384E 03	-0.91020E-01
0.33367E 03	0.33370E 03	-0.98227E-02
0.37357E 03	0.37378E 03	-0.57167E-01
0.41357E 03	0.41418E 03	-0.14753E 00
0.47357E 03	0.47365E 03	-0.16973E-01
0.56157E 03	0.56190E 03	-0.59255E-01
0.66357E 03	0.66318E 03	0.58149E-01

TABLE D-38

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 800 PSIG
TEST-METHOD USED = CASE-1
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-2
RUN NUMBER = 4

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.175714E 02	0.136421E 02	0.235827E-03	0.176109E-01	0.933193E 01
0.190448E 02	0.136421E 02	0.333442E-03	0.176117E-01	0.933227E 01
0.210095E 02	0.136421E 02	0.470356E-03	0.176242E-01	0.934672E 01
0.234653E 02	0.136421E 02	0.652965E-03	0.176255E-01	0.934728E 01
0.259211E 02	0.136421E 02	0.848655E-03	0.176268E-01	0.934785E 01
0.283769E 02	0.136421E 02	0.106122E-02	0.176282E-01	0.934841E 01
0.322939E 02	0.136439E 02	0.143209E-02	0.176246E-01	0.934231E 01
0.372177E 02	0.136457E 02	0.193401E-02	0.176273E-01	0.934344E 01
0.421171E 02	0.136484E 02	0.248994E-02	0.176299E-01	0.934456E 01
0.476421E 02	0.136502E 02	0.321423E-02	0.176329E-01	0.934583E 01
0.536421E 02	0.136529E 02	0.405889E-02	0.176362E-01	0.934721E 01
0.616324E 02	0.136469E 02	0.530235E-02	0.176544E-01	0.936622E 01
0.736324E 02	0.146541E 02	0.739049E-02	0.176615E-01	0.936923E 01
0.876324E 02	0.136649E 02	0.103412E-01	0.176686E-01	0.937225E 01
0.103632E 03	0.136776E 02	0.142123E-01	0.176773E-01	0.937598E 01
0.123632E 03	0.137010E 02	0.197556E-01	0.176882E-01	0.938066E 01
0.143782E 03	0.138961E 02	0.263307E-01	0.176993E-01	0.938542E 01
0.163632E 03	0.139358E 02	0.336414E-01	0.177102E-01	0.939010E 01
0.193632E 03	0.140152E 02	0.463612E-01	0.177267E-01	0.939720E 01
0.223632E 03	0.141164E 02	0.613307E-01	0.177432E-01	0.940434E 01
0.253532E 03	0.142536E 02	0.779469E-01	0.177680E-01	0.942179E 01
0.293332E 03	0.144674E 02	0.103676E 00	0.177902E-01	0.943144E 01
0.333732E 03	0.148358E 02	0.133613E 00	0.178129E-01	0.944136E 01
0.373632E 03	0.153024E 02	0.166343E 00	0.178355E-01	0.945127E 01
0.433622E 03	0.162505E 02	0.221191E 00	0.178446E-01	0.943444E 01
0.522623E 03	0.184361E 02	0.320502E 00	0.178968E-01	0.945738E 01
0.657623E 03	0.235442E 02	0.502547E 00	0.179788E-01	0.949381E 01
0.840623E 03	0.340059E 02	0.801225E 00	0.180964E-01	0.954684E 01
0.101962E 04	0.488228E 02	0.116503E 01	0.182205E-01	0.960370E 01

TABLE D-39

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.6960E 00	0.8376E 01	0.1857E 10
1	0.6032E 00	0.1007E 02	0.9028E 09
2	0.6116E 00	0.1015E 02	0.8365E 09
3	0.6117E 00	0.1015E 02	0.8368E 09
4	0.6117E 00	0.1015E 02	0.8368E 09
5	0.6117E 00	0.1015E 02	0.8368E 09

FRACTIONAL POROSITY = 0.1984E 00

SUM OF ERROR SQUARES = 0.1056E 02
VARIANCE = 0.4063E 00
STANDARD DEVIATION = 0.6374E 00

VARIANCE-COVARIANCE MATRIX

0.23280E-05	-0.41027E-03	0.23304E 06
-0.41027E-03	0.99354E-01	-0.35968E 08
0.23304E 06	-0.35968E 08	0.28716E 17

CORRELATION MATRIX

1.00000	-0.85308	0.90130
-0.85308	1.00000	-0.67338
0.90130	-0.67338	1.00000

TABLE D-40

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.17571E 02	0.17651E 02	-0.45620E 00
0.19044E 02	0.19147E 02	-0.53928E 00
0.21009E 02	0.21131E 02	-0.58087E 00
0.23465E 02	0.23577E 02	-0.47841E 00
0.25921E 02	0.26013E 02	-0.35534E 00
0.28376E 02	0.28499E 02	-0.43118E 00
0.32293E 02	0.32485E 02	-0.59383E 00
0.37217E 02	0.37322E 02	-0.28217E 00
0.42117E 02	0.42170E 02	-0.12794E 00
0.47642E 02	0.47937E 02	-0.61930E 00
0.53642E 02	0.53973E 02	-0.61756E 00
0.61632E 02	0.62001E 02	-0.59856E 00
0.73632E 02	0.73999E 02	-0.49894E 00
0.87632E 02	0.87895E 02	-0.29989E 00
0.10363E 03	0.10388E 03	-0.23916E 00
0.12363E 03	0.12346E 03	0.13331E 00
0.14378E 03	0.14356E 03	0.14859E 00
0.16363E 03	0.16318E 03	0.27298E 00
0.19363E 03	0.19288E 03	0.38488E 00
0.22363E 03	0.22311E 03	0.23290E 00
0.25353E 03	0.25279E 03	0.29240E 00
0.29333E 03	0.29301E 03	0.10915E 00
0.33373E 03	0.33404E 03	-0.92760E-01
0.37363E 03	0.37399E 03	-0.96968E-01
0.43362E 03	0.43234E 03	0.29560E 00
0.52262E 03	0.52330E 03	-0.13021E 00
0.65762E 03	0.65970E 03	-0.31702E 00
0.84062E 03	0.83943E 03	0.14117E 00
0.10196E 04	0.10197E 04	-0.93621E-02

TABLE D-41

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 1000 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-2
RUN NUMBER = 5

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.185343E 02	0.136228E 02	0.293275E-03	0.176367E-01	0.936326E 01
0.195167E 02	0.136228E 02	0.358208E-03	0.176373E-01	0.936349E 01
0.209901E 02	0.136228E 02	0.458199E-03	0.176463E-01	0.937395E 01
0.234459E 02	0.136228E 02	0.635052E-03	0.176476E-01	0.937452E 01
0.259017E 02	0.136228E 02	0.826222E-03	0.176489E-01	0.937509E 01
0.283575E 02	0.136228E 02	0.103135E-02	0.176503E-01	0.937565E 01
0.313045E 02	0.136246E 02	0.128979E-02	0.176519E-01	0.937633E 01
0.352338E 02	0.136264E 02	0.165673E-02	0.176540E-01	0.937725E 01
0.411277E 02	0.136291E 02	0.228550E-02	0.176572E-01	0.937861E 01
0.476228E 02	0.136318E 02	0.304555E-02	0.176746E-01	0.939728E 01
0.536228E 02	0.136336E 02	0.384058E-02	0.176779E-01	0.939868E 01
0.636228E 02	0.136372E 02	0.533096E-02	0.176833E-01	0.940101E 01
0.736228E 02	0.136444E 02	0.701006E-02	0.176888E-01	0.940336E 01
0.876131E 02	0.136420E 02	0.972788E-02	0.177160E-01	0.943087E 01
0.103613E 03	0.136564E 02	0.132825E-01	0.177247E-01	0.943465E 01
0.123413E 03	0.136781E 02	0.184467E-01	0.177355E-01	0.943935E 01
0.143663E 03	0.136572E 02	0.246552E-01	0.176907E-01	0.937483E 01
0.163613E 03	0.136572E 02	0.315509E-01	0.177016E-01	0.937950E 01
0.193513E 03	0.138739E 02	0.435439E-01	0.177348E-01	0.940737E 01
0.223613E 03	0.139660E 02	0.565359E-01	0.177514E-01	0.941456E 01
0.253584E 03	0.140969E 02	0.719512E-01	0.177935E-01	0.945365E 01
0.293384E 03	0.142991E 02	0.944518E-01	0.178157E-01	0.946338E 01
0.333584E 03	0.145910E 02	0.121455E 00	0.178382E-01	0.947330E 01
0.373584E 03	0.149593E 02	0.152433E 00	0.178608E-01	0.948329E 01
0.433584E 03	0.157943E 02	0.200664E 00	0.179173E-01	0.952640E 01
0.508584E 03	0.173169E 02	0.274250E 00	0.179610E-01	0.954603E 01
0.638084E 03	0.213198E 02	0.429070E 00	0.180388E-01	0.958128E 01
0.788584E 03	0.284171E 02	0.648968E 00	0.181337E-01	0.962478E 01
0.890084E 03	0.342864E 02	0.819749E 00	0.182005E-01	0.965575E 01

TABLE D-42

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.6960E 00	0.8376E 01	0.1857E 10
1	0.5212E 00	0.1296E 02	-0.8179E 09
2	0.5485E 00	0.1336E 02	-0.1124E 10
3	0.5496E 00	0.1334E 02	-0.1114E 10
4	0.5496E 00	0.1334E 02	-0.1114E 10
5	0.5496E 00	0.1334E 02	-0.1114E 10

FRACTIONAL POROSITY = 0.1391E 00

SUM OF ERROR SQUARES = 0.9009E 01
VARIANCE = 0.3465E 00
STANDARD DEVIATION = 0.5886E 00

VARIANCE-COVARIANCE MATRIX

0.20187E-05	-0.38384E-03	0.34654E 06
-0.38384E-03	0.96823E-01	-0.58026E 08
0.34654E 06	-0.58026E 08	0.71876E 17

CORRELATION MATRIX

1.00000	-0.86821	0.90978
-0.86821	1.00000	-0.69557
0.90978	-0.69557	1.00000

TABLE D-43

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.18534E 02	0.18426E 02	0.58182E 00
0.19516E 02	0.19402E 02	0.58750E 00
0.20990E 02	0.20851E 02	0.65926E 00
0.23445E 02	0.23267E 02	0.76123E 00
0.25901E 02	0.25727E 02	0.67248E 00
0.28357E 02	0.28214E 02	0.50522E 00
0.31304E 02	0.31130E 02	0.55636E 00
0.35233E 02	0.34951E 02	0.80088E 00
0.41127E 02	0.40948E 02	0.43519E 00
0.47622E 02	0.47427E 02	0.40962E 00
0.53622E 02	0.53529E 02	0.17447E 00
0.63622E 02	0.63669E 02	-0.74084E-01
0.73622E 02	0.73670E 02	-0.65202E-01
0.87613E 02	0.87943E 02	-0.37650E 00
0.10361E 03	0.10388E 03	-0.26080E 00
0.12341E 03	0.12381E 03	-0.32176E 00
0.14366E 03	0.14388E 03	-0.15273E 00
0.16361E 03	0.16398E 03	-0.22628E 00
0.19351E 03	0.19466E 03	-0.59624E 00
0.22361E 03	0.22317E 03	0.19404E 00
0.25358E 03	0.25366E 03	-0.32625E-01
0.29338E 03	0.29227E 03	0.37962E 00
0.33358E 03	0.33307E 03	0.15257E 00
0.37358E 03	0.37469E 03	-0.29867E 00
0.43358E 03	0.43242E 03	0.26626E 00
0.50858E 03	0.50788E 03	0.13814E 00
0.63808E 03	0.63896E 03	-0.13789E 00
0.78858E 03	0.78946E 03	-0.11230E 00
0.89008E 03	0.88937E 03	0.80119E-01

TABLE D-44

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 400 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-3-A
RUN NUMBER = 1

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.145374E 02	0.135605E 02	0.453921E-03	0.175328E-01	0.923732E 01
0.150286E 02	0.135614E 02	0.692245E-03	0.175331E-01	0.923743E 01
0.155197E 02	0.135614E 02	0.937754E-03	0.175333E-01	0.923754E 01
0.160205E 02	0.135720E 02	0.118878E-02	0.175450E-01	0.925160E 01
0.165117E 02	0.135720E 02	0.144822E-02	0.175452E-01	0.925171E 01
0.174940E 02	0.135729E 02	0.199024E-02	0.175458E-01	0.925193E 01
0.189675E 02	0.135756E 02	0.285460E-02	0.175466E-01	0.925225E 01
0.209321E 02	0.135783E 02	0.410262E-02	0.175502E-01	0.925579E 01
0.233879E 02	0.135828E 02	0.581691E-02	0.175515E-01	0.925634E 01
0.263349E 02	0.135900E 02	0.812240E-02	0.175531E-01	0.925699E 01
0.302738E 02	0.136105E 02	0.115659E-01	0.175641E-01	0.926874E 01
0.351854E 02	0.136268E 02	0.165007E-01	0.175668E-01	0.926985E 01
0.410793E 02	0.136539E 02	0.233571E-01	0.175700E-01	0.927117E 01
0.476880E 02	0.137794E 02	0.321423E-01	0.175850E-01	0.928668E 01
0.536880E 02	0.138299E 02	0.412882E-01	0.175883E-01	0.928805E 01
0.615938E 02	0.139260E 02	0.548975E-01	0.175981E-01	0.929655E 01
0.715938E 02	0.139964E 02	0.745944E-01	0.176035E-01	0.929883E 01
0.835938E 02	0.143196E 02	0.101727E 00	0.176102E-01	0.930162E 01
0.976073E 02	0.147862E 02	0.137950E 00	0.176238E-01	0.931196E 01
0.113407E 03	0.154663E 02	0.184149E 00	0.176328E-01	0.931572E 01
0.133707E 03	0.167262E 02	0.252025E 00	0.176446E-01	0.932066E 01
0.153613E 03	0.184019E 02	0.326344E 00	0.176591E-01	0.932909E 01
0.183613E 03	0.216435E 02	0.449423E 00	0.176774E-01	0.933679E 01
0.213513E 03	0.257079E 02	0.582585E 00	0.176960E-01	0.934471E 01
0.253222E 03	0.322131E 02	0.768602E 00	0.177216E-01	0.935557E 01
0.293422E 03	0.399243E 02	0.986276E 00	0.177482E-01	0.936696E 01
0.343772E 03	0.506228E 02	0.127148E 01	0.177823E-01	0.938165E 01

TABLE D-45

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.8577E 01	0.4938E 01	0.2655E 10
1	0.7763E 01	0.1417E 01	0.1747E 10
2	0.7868E 01	0.9613E 00	0.1756E 10
3	0.7869E 01	0.9621E 00	0.1756E 10
4	0.7869E 01	0.9621E 00	0.1756E 10

FRACTIONAL POROSITY = 0.6181E 00

SUM OF ERROR SQUARES = 0.3959E 01
VARIANCE = 0.1649E 00
STANDARD DEVIATION = 0.4061E 00

VARIANCE-COVARIANCE MATRIX

0.38489E-02	-0.21608E-01	0.19148E 07
-0.21608E-01	0.14261E 00	-0.96168E 07
0.19148E 07	-0.96168E 07	0.11322E 16

CORRELATION MATRIX

1.00000	-0.92231	0.91724
-0.92231	1.00000	-0.75679
0.91724	-0.75679	1.00000

TABLE D-46

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14537E 02	0.14550E 02	-0.92012E-01
0.15028E 02	0.15047E 02	-0.12286E 00
0.15519E 02	0.15541E 02	-0.14234E 00
0.16020E 02	0.16045E 02	-0.15479E 00
0.16511E 02	0.16538E 02	-0.16410E 00
0.17494E 02	0.17528E 02	-0.19644E 00
0.18967E 02	0.19006E 02	-0.20580E 00
0.20932E 02	0.20967E 02	-0.17057E 00
0.23387E 02	0.23406E 02	-0.78942E-01
0.26334E 02	0.26349E 02	-0.56116E-01
0.30273E 02	0.30257E 02	0.53288E-01
0.35185E 02	0.35122E 02	0.17973E 00
0.41079E 02	0.40976E 02	0.25028E 00
0.47688E 02	0.47544E 02	0.30077E 00
0.53688E 02	0.53535E 02	0.28330E 00
0.61593E 02	0.61472E 02	0.19678E 00
0.71593E 02	0.71493E 02	0.13974E 00
0.83593E 02	0.83565E 02	0.33677E-01
0.97607E 02	0.97680E 02	-0.74834E-01
0.11340E 03	0.11350E 03	-0.83635E-01
0.13370E 03	0.13403E 03	-0.24593E 00
0.15361E 03	0.15418E 03	-0.37370E 00
0.18361E 03	0.18415E 03	-0.29578E 00
0.21351E 03	0.21362E 03	-0.50254E-01
0.25322E 03	0.25164E 03	0.62497E 00
0.29342E 03	0.29312E 03	0.10169E 00
0.34377E 03	0.34449E 03	-0.20902E 00

TABLE D-47

A SEMI-GRAPHICAL METHOD OF PARAMETER ESTIMATION
I.E. A SIMULTANEOUS LINEAR LEAST SQUARES FIT TECHNIQUE

NV	B(NV)	KV(NV)	KI(NI)	FB(NI)	NRE(NV)
4	0.2028E 01	0.7444E 01	0.7565E 01	0.1529E 10	0.3375E-03
5	0.2013E 01	0.7451E 01	0.7577E 01	0.1541E 10	0.4136E-03
6	0.1781E 01	0.7558E 01	0.7646E 01	0.1595E 10	0.5866E-03
7	0.1674E 01	0.7607E 01	0.7678E 01	0.1621E 10	0.8514E-03
8	0.1691E 01	0.7600E 01	0.7680E 01	0.1624E 10	0.1219E-02
9	0.1762E 01	0.7567E 01	0.7671E 01	0.1620E 10	0.1713E-02
10	0.1741E 01	0.7576E 01	0.7684E 01	0.1632E 10	0.2394E-02
11	0.1751E 01	0.7572E 01	0.7692E 01	0.1641E 10	0.3400E-02
12	0.1775E 01	0.7561E 01	0.7699E 01	0.1650E 10	0.4838E-02
13	0.1790E 01	0.7555E 01	0.7710E 01	0.1662E 10	0.6845E-02
14	0.1804E 01	0.7549E 01	0.7723E 01	0.1676E 10	0.9422E-02
15	0.1812E 01	0.7546E 01	0.7739E 01	0.1692E 10	0.1215E-01
16	0.1814E 01	0.7545E 01	0.7756E 01	0.1708E 10	0.1621E-01
17	0.1831E 01	0.7538E 01	0.7771E 01	0.1723E 10	0.2208E-01
18	0.1863E 01	0.7525E 01	0.7783E 01	0.1736E 10	0.3014E-01
19	0.1919E 01	0.7503E 01	0.7790E 01	0.1745E 10	0.4079E-01
20	0.2011E 01	0.7467E 01	0.7795E 01	0.1754E 10	0.5431E-01
21	0.2137E 01	0.7419E 01	0.7782E 01	0.1750E 10	0.7345E-01
22	0.2293E 01	0.7361E 01	0.7729E 01	0.1717E 10	0.9235E-01
23	0.2525E 01	0.7278E 01	0.7610E 01	0.1637E 10	0.1196E 00

DATA DISCRIMINATION METHODS

METHOD 1. FINDING THE BEST FIT FOR THE KLINKENBERG PLOT
I.E. THE MAXIMUM CORRELATION COEFFICIENT
NV= 16 R(NV)= 0.999 (MAXIMUM)
NI= 11 R(NI)= 0.997

METHOD 2. MATCHING THE VALUES OF K FROM THE KLINKENBERG AND MODIFIED VISCO-INERTIAL PLOTS
I.E. THE MINIMUM RELATIVE ERROR BETWEEN KV AND KI
NV= 7
NI= 20
ERRK(NV)=-0.9361E 00 (MIN. PERCENT)

METHOD 3. THE REYNOLDS NUMBER CRITERION, I.E. THE POINTS ARE IN VISCOUS-REGION IF NRE IS LESS THAN 0.01
NV= 14
NI= 13
NRE(NV)= 0.9422E-02

TABLE D-48

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 600 PSIG
TEST-METHOD USED = CASE-1
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-3-A
RUN NUMBER = 2

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.145544E 02	0.137249E 02	0.351849E-03	0.175525E-01	0.926139E 01
0.152420E 02	0.137249E 02	0.655784E-03	0.175529E-01	0.926154E 01
0.162244E 02	0.137267E 02	0.110867E-02	0.175534E-01	0.926176E 01
0.177075E 02	0.137381E 02	0.183117E-02	0.175599E-01	0.926908E 01
0.191319E 02	0.137399E 02	0.257701E-02	0.175607E-01	0.926940E 01
0.210965E 02	0.137417E 02	0.368683E-02	0.175617E-01	0.926984E 01
0.236014E 02	0.137472E 02	0.523265E-02	0.175688E-01	0.927738E 01
0.264993E 02	0.137526E 02	0.719154E-02	0.175703E-01	0.927803E 01
0.305022E 02	0.137616E 02	0.102361E-01	0.175725E-01	0.927893E 01
0.353892E 02	0.137779E 02	0.144795E-01	0.175752E-01	0.928003E 01
0.413077E 02	0.137995E 02	0.203152E-01	0.175841E-01	0.928836E 01
0.478291E 02	0.138916E 02	0.277686E-01	0.175917E-01	0.929484E 01
0.577291E 02	0.139657E 02	0.408454E-01	0.175971E-01	0.929709E 01
0.697291E 02	0.140975E 02	0.597847E-01	0.176111E-01	0.930888E 01
0.837791E 02	0.143250E 02	0.861497E-01	0.176188E-01	0.931214E 01
0.998485E 02	0.146766E 02	0.121308E 00	0.176644E-01	0.936125E 01
0.117748E 03	0.152710E 02	0.166942E 00	0.176745E-01	0.936555E 01
0.137748E 03	0.163025E 02	0.225379E 00	0.176860E-01	0.937046E 01
0.157748E 03	0.176777E 02	0.291227E 00	0.176977E-01	0.937546E 01
0.183748E 03	0.199616E 02	0.384533E 00	0.177132E-01	0.938210E 01
0.213748E 03	0.232524E 02	0.499772E 00	0.177315E-01	0.938997E 01
0.253348E 03	0.286797E 02	0.659835E 00	0.177620E-01	0.940782E 01
0.293548E 03	0.352612E 02	0.851400E 00	0.177880E-01	0.941909E 01
0.343748E 03	0.442485E 02	0.110897E 01	0.178211E-01	0.943353E 01
0.412748E 03	0.594985E 02	0.148835E 01	0.178687E-01	0.945443E 01

TABLE D-49

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.7869E 01	0.9621E 00	0.1756E 10
1	0.6040E 01	0.3041E 01	0.1832E 10
2	0.6310E 01	0.3234E 01	0.1824E 10
3	0.6319E 01	0.3228E 01	0.1824E 10
4	0.6319E 01	0.3228E 01	0.1824E 10

FRACTIONAL POROSITY = 0.2683E 00

SUM OF ERROR SQUARES = 0.1178E 02
VARIANCE = 0.5358E 00
STANDARD DEVIATION = 0.7319E 00

VARIANCE-COVARIANCE MATRIX

0.49102E-02	-0.41143E-01	0.33035E 07
-0.41143E-01	0.41191E 00	-0.24460E 08
0.33035E 07	-0.24460E 08	0.26896E 16

CORRELATION MATRIX

1.00000	-0.91484	0.90902
-0.91484	1.00000	-0.73487
0.90902	-0.73487	1.00000

TABLE D-50

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14554E 02	0.14552E 02	0.13471E-01
0.15242E 02	0.15237E 02	0.29757E-01
0.16224E 02	0.16215E 02	0.56405E-01
0.17707E 02	0.17690E 02	0.95219E-01
0.19131E 02	0.19102E 02	0.15352E 00
0.21096E 02	0.21052E 02	0.20709E 00
0.23601E 02	0.23543E 02	0.24357E 00
0.26499E 02	0.26395E 02	0.39269E 00
0.30502E 02	0.30358E 02	0.47229E 00
0.35389E 02	0.35219E 02	0.48088E 00
0.41307E 02	0.41067E 02	0.58279E 00
0.47829E 02	0.47617E 02	0.44186E 00
0.57729E 02	0.57428E 02	0.52083E 00
0.69729E 02	0.69459E 02	0.38704E 00
0.83779E 02	0.83604E 02	0.20890E 00
0.99848E 02	0.99958E 02	-0.11050E 00
0.11774E 03	0.11809E 03	-0.29514E 00
0.13774E 03	0.13846E 03	-0.51724E 00
0.15774E 03	0.15893E 03	-0.75322E 00
0.18374E 03	0.18504E 03	-0.70696E 00
0.21374E 03	0.21424E 03	-0.23069E 00
0.25334E 03	0.25134E 03	0.78929E 00
0.29354E 03	0.29217E 03	0.46832E 00
0.34374E 03	0.34324E 03	0.14522E 00
0.41274E 03	0.41389E 03	-0.27658E 00

TABLE D-51

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 800 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-3-A
RUN NUMBER = 3

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.145604E 02	0.136572E 02	0.351719E-03	0.176314E-01	0.935846E 01
0.160584E 02	0.136590E 02	0.972781E-03	0.176323E-01	0.935881E 01
0.176450E 02	0.136511E 02	0.168378E-02	0.176388E-01	0.936617E 01
0.191431E 02	0.136511E 02	0.240199E-02	0.176396E-01	0.936652E 01
0.211030E 02	0.136451E 02	0.340966E-02	0.176494E-01	0.937771E 01
0.235047E 02	0.136487E 02	0.475847E-02	0.176507E-01	0.937827E 01
0.264026E 02	0.136523E 02	0.654290E-02	0.176523E-01	0.937894E 01
0.303318E 02	0.136613E 02	0.925098E-02	0.176544E-01	0.937986E 01
0.352434E 02	0.136740E 02	0.130819E-01	0.176571E-01	0.938100E 01
0.412505E 02	0.136860E 02	0.184452E-01	0.176687E-01	0.939268E 01
0.478131E 02	0.137955E 02	0.251236E-01	0.176748E-01	0.939736E 01
0.577073E 02	0.138511E 02	0.369591E-01	0.176834E-01	0.940358E 01
0.696073E 02	0.139612E 02	0.538583E-01	0.176899E-01	0.940639E 01
0.836073E 02	0.141544E 02	0.774867E-01	0.176976E-01	0.940972E 01
0.996034E 02	0.145334E 02	0.109258E 00	0.177122E-01	0.942062E 01
0.117803E 03	0.149909E 02	0.150864E 00	0.177224E-01	0.942502E 01
0.137603E 03	0.158382E 02	0.203059E 00	0.177337E-01	0.942991E 01
0.157609E 03	0.170170E 02	0.261958E 00	0.177452E-01	0.943494E 01
0.183603E 03	0.190062E 02	0.346104E 00	0.177606E-01	0.944162E 01
0.213403E 03	0.219040E 02	0.455106E 00	0.177868E-01	0.945970E 01
0.253403E 03	0.268279E 02	0.613583E 00	0.178116E-01	0.947059E 01
0.293603E 03	0.326850E 02	0.783949E 00	0.178371E-01	0.948185E 01
0.343503E 03	0.413044E 02	0.101104E 01	0.178699E-01	0.949633E 01
0.412403E 03	0.546034E 02	0.137172E 01	0.179163E-01	0.951702E 01

TABLE D-52

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.6319E 01	0.3227E 01	0.1824E 10
1	0.5695E 01	0.3333E 01	0.2027E 10
2	0.5737E 01	0.3346E 01	0.2032E 10
3	0.5737E 01	0.3346E 01	0.2032E 10
4	0.5737E 01	0.3346E 01	0.2032E 10

FRACTIONAL POROSITY = 0.2455E 00

SUM OF ERROR SQUARES = 0.6563E 01
VARIANCE = 0.3125E 00
STANDARD DEVIATION = 0.5590E 00

VARIANCE-COVARIANCE MATRIX

0.23190E-02	-0.21576E-01	0.20593E 07
-0.21576E-01	0.23954E 00	-0.16957E 08
0.20593E 07	-0.16957E 08	0.22072E 16

CORRELATION MATRIX

1.00000	-0.91543	0.91023
-0.91543	1.00000	-0.73747
0.91023	-0.73747	1.00000

TABLE D-53

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14560E 02	0.14576E 02	-0.10927E 00
0.16058E 02	0.16094E 02	-0.22191E 00
0.17645E 02	0.17691E 02	-0.26076E 00
0.19143E 02	0.19193E 02	-0.26561E 00
0.21103E 02	0.21150E 02	-0.22358E 00
0.23504E 02	0.23549E 02	-0.19231E 00
0.26402E 02	0.26429E 02	-0.10034E 00
0.30331E 02	0.30338E 02	-0.23128E-01
0.35243E 02	0.35205E 02	0.10901E 00
0.41250E 02	0.41166E 02	0.20474E 00
0.47813E 02	0.47666E 02	0.30774E 00
0.57707E 02	0.57515E 02	0.33208E 00
0.69607E 02	0.69381E 02	0.32390E 00
0.83607E 02	0.83436E 02	0.20473E 00
0.99603E 02	0.99613E 02	-0.10509E-01
0.11780E 03	0.11783E 03	-0.25581E-01
0.13760E 03	0.13785E 03	-0.18132E 00
0.15760E 03	0.15799E 03	-0.24822E 00
0.18360E 03	0.18384E 03	-0.13036E 00
0.21340E 03	0.21411E 03	-0.33465E 00
0.25340E 03	0.25379E 03	-0.15569E 00
0.29360E 03	0.29294E 03	0.22393E 00
0.34350E 03	0.34160E 03	0.55355E 00
0.41240E 03	0.41359E 03	-0.28868E 00

TABLE D-54

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 1000 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = LS-3-A
RUN NUMBER = 4

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.145567E 02	0.135762E 02	0.353195E-03	0.176402E-01	0.936934E 01
0.160548E 02	0.135780E 02	0.928385E-03	0.176410E-01	0.936968E 01
0.175431E 02	0.135702E 02	0.154537E-02	0.176642E-01	0.939758E 01
0.189184E 02	0.135720E 02	0.214903E-02	0.176650E-01	0.939790E 01
0.210009E 02	0.135738E 02	0.313125E-02	0.176746E-01	0.940883E 01
0.233879E 02	0.135774E 02	0.436353E-02	0.176759E-01	0.940938E 01
0.282504E 02	0.135864E 02	0.722236E-02	0.176785E-01	0.941052E 01
0.332014E 02	0.135912E 02	0.105846E-01	0.176895E-01	0.942197E 01
0.411091E 02	0.136093E 02	0.169711E-01	0.176938E-01	0.942383E 01
0.496551E 02	0.137212E 02	0.251986E-01	0.176985E-01	0.942586E 01
0.597648E 02	0.137905E 02	0.368743E-01	0.176704E-01	0.938673E 01
0.716648E 02	0.138970E 02	0.531099E-01	0.176770E-01	0.938953E 01
0.856648E 02	0.140703E 02	0.756980E-01	0.177015E-01	0.941363E 01
0.101564E 03	0.143520E 02	0.105833E 00	0.177103E-01	0.941743E 01
0.119464E 03	0.148172E 02	0.145053E 00	0.177203E-01	0.942177E 01
0.139668E 03	0.155824E 02	0.194406E 00	0.177347E-01	0.943034E 01
0.163568E 03	0.168717E 02	0.259662E 00	0.177484E-01	0.943633E 01
0.189568E 03	0.187012E 02	0.339427E 00	0.177637E-01	0.944298E 01
0.219424E 03	0.214084E 02	0.440111E 00	0.177872E-01	0.945773E 01
0.253574E 03	0.252149E 02	0.562859E 00	0.178081E-01	0.946692E 01
0.293574E 03	0.304703E 02	0.723786E 00	0.178332E-01	0.947798E 01
0.343374E 03	0.382797E 02	0.937421E 00	0.178654E-01	0.949222E 01
0.410574E 03	0.500744E 02	0.125888E 01	0.179100E-01	0.951206E 01

TABLE D-55

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.5737E 01	0.3345E 01	0.2032E 10
1	0.5233E 01	0.3912E 01	0.2367E 10
2	0.5265E 01	0.3928E 01	0.2379E 10
3	0.5265E 01	0.3927E 01	0.2379E 10
4	0.5265E 01	0.3927E 01	0.2379E 10

FRACTIONAL POROSITY = 0.2006E 00

SUM OF ERROR SQUARES = 0.5675E 01
VARIANCE = 0.2837E 00
STANDARD DEVIATION = 0.5327E 00

VARIANCE-COVARIANCE MATRIX

0.18556E-02	-0.19434E-01	0.20896E 07
-0.19434E-01	0.24019E 00	-0.19389E 08
0.20896E 07	-0.19389E 08	0.28394E 16

CORRELATION MATRIX

1.00000	-0.92053	0.91035
-0.92053	1.00000	-0.74243
0.91035	-0.74243	1.00000

TABLE D-56

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14556E 02	0.14548E 02	0.55346E-01
0.16054E 02	0.16033E 02	0.13460E 00
0.17543E 02	0.17516E 02	0.15271E 00
0.18918E 02	0.18874E 02	0.23351E 00
0.21000E 02	0.20933E 02	0.32089E 00
0.23387E 02	0.23295E 02	0.39663E 00
0.28250E 02	0.28119E 02	0.46492E 00
0.33201E 02	0.33015E 02	0.56156E 00
0.41109E 02	0.40877E 02	0.56389E 00
0.49655E 02	0.49385E 02	0.54246E 00
0.59764E 02	0.59430E 02	0.55976E 00
0.71664E 02	0.71397E 02	0.37297E 00
0.85664E 02	0.85665E 02	-0.12112E-02
0.10156E 03	0.10188E 03	-0.31908E 00
0.11946E 03	0.12018E 03	-0.60040E 00
0.13966E 03	0.14039E 03	-0.52284E 00
0.16356E 03	0.16405E 03	-0.29857E 00
0.18956E 03	0.18995E 03	-0.20359E 00
0.21942E 03	0.21965E 03	-0.10521E 00
0.25357E 03	0.25274E 03	0.32792E 00
0.29357E 03	0.29280E 03	0.26114E 00
0.34337E 03	0.34227E 03	0.31918E 00
0.41057E 03	0.41168E 03	-0.26939E 00

TABLE D-57

FLOW RATE PREDICTIONS

USING THE ESTIMATED PARAMETERS K,B,AND FB

WHERE K= 0.5265E 01 MDS
B= 0.3927E 01 PSIA
FB= 0.2379E 10 1/FT

I	QO(OBS.)	QO(PRED.)	PERCENT ERROR
1	0.353195E-03	0.355304E-03	-0.597378E 00
2	0.928385E-03	0.934809E-03	-0.691945E 00
3	0.154537E-02	0.155437E-02	-0.582856E 00
4	0.214903E-02	0.216516E-02	-0.750703E 00
5	0.313125E-02	0.315890E-02	-0.883213E 00
6	0.722236E-02	0.729537E-02	-0.101087E 01
7	0.436353E-02	0.440620E-02	-0.977951E 00
8	0.105846E-01	0.107062E-01	-0.114921E 01
9	0.169711E-01	0.171581E-01	-0.110201E 01
10	0.251986E-01	0.254600E-01	-0.103735E 01
11	0.368743E-01	0.372644E-01	-0.105813E 01
12	0.531099E-01	0.534799E-01	-0.696745E 00
13	0.756980E-01	0.756915E-01	0.854329E-02
14	0.105833E 00	0.105200E 00	0.597747E 00
15	0.145053E 00	0.143446E 00	0.110723E 01
16	0.194406E 00	0.192543E 00	0.958259E 00
17	0.259662E 00	0.258247E 00	0.544761E 00
18	0.339427E 00	0.338178E 00	0.367784E 00
19	0.440111E 00	0.439283E 00	0.187937E 00
20	0.562859E 00	0.566068E 00	-0.570145E 00
21	0.723786E 00	0.727001E 00	-0.444218E 00
22	0.937421E 00	0.942390E 00	-0.530084E 00
23	0.125888E 01	0.125352E 01	0.425179E 00

TABLE D-58

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 400 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = SS-1-A
RUN NUMBER = 1

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.636244E 02	0.135744E 02	0.220636E-03	0.176273E-01	0.933186E 01
0.675744E 02	0.135744E 02	0.243359E-03	0.176633E-01	0.937460E 01
0.735744E 02	0.135744E 02	0.279473E-03	0.176780E-01	0.939005E 01
0.815744E 02	0.135744E 02	0.331768E-03	0.176823E-01	0.939192E 01
0.915744E 02	0.135744E 02	0.403745E-03	0.176877E-01	0.939426E 01
0.101574E 03	0.135744E 02	0.481426E-03	0.176932E-01	0.939660E 01
0.113574E 03	0.135744E 02	0.582830E-03	0.176966E-01	0.939550E 01
0.127574E 03	0.135656E 02	0.706144E-03	0.177042E-01	0.939879E 01
0.143664E 03	0.135666E 02	0.873901E-03	0.177016E-01	0.938845E 01
0.163564E 03	0.135684E 02	0.109303E-02	0.177125E-01	0.939314E 01
0.183564E 03	0.135693E 02	0.133196E-02	0.177235E-01	0.939786E 01
0.203364E 03	0.135702E 02	0.160300E-02	0.177343E-01	0.940255E 01
0.233364E 03	0.135711E 02	0.205537E-02	0.177653E-01	0.942768E 01
0.263414E 03	0.135740E 02	0.256205E-02	0.177818E-01	0.943490E 01
0.293314E 03	0.135738E 02	0.311914E-02	0.177984E-01	0.944214E 01
0.323464E 03	0.135756E 02	0.379890E-02	0.178152E-01	0.944948E 01
0.363464E 03	0.135792E 02	0.495052E-02	0.178459E-01	0.946979E 01
0.403164E 03	0.135828E 02	0.615770E-02	0.178682E-01	0.947966E 01
0.453564E 03	0.135882E 02	0.814346E-02	0.178968E-01	0.949232E 01
0.535564E 03	0.136153E 02	0.152916E-01	0.179438E-01	0.951328E 01

TABLE D-59

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.1250E-01	0.6308E 02	0.3229E 13
1	0.1466E-01	0.5227E 02	-0.7572E 14
2	0.1548E-01	0.4903E 02	-0.5565E 14
3	0.1566E-01	0.4804E 02	-0.5376E 14
4	0.1566E-01	0.4804E 02	-0.5375E 14
5	0.1566E-01	0.4804E 02	-0.5375E 14

FRACTIONAL POROSITY = 0.9244E-04

SUM OF ERROR SQUARES = 0.2823E 02
VARIANCE = 0.1661E 01
STANDARD DEVIATION = 0.1288E 01

VARIANCE-COVARIANCE MATRIX

0.13819E-07	-0.12456E-03	0.90514E 08
-0.12456E-03	0.12500E 01	-0.77014E 12
0.90514E 08	-0.77014E 12	0.67226E 24

CORRELATION MATRIX

1.00000	-0.94771	0.93906
-0.94771	1.00000	-0.84010
0.93906	-0.84010	31025.89758

TABLE D-60

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.63624E 02	0.62497E 02	0.17714E 01
0.67574E 02	0.66646E 02	0.13726E 01
0.73574E 02	0.72618E 02	0.12996E 01
0.81574E 02	0.80736E 02	0.10272E 01
0.91574E 02	0.91212E 02	0.39537E 00
0.10157E 03	0.10162E 03	-0.50129E-01
0.11357E 03	0.11411E 03	-0.47335E 00
0.12757E 03	0.12780E 03	-0.17939E 00
0.14366E 03	0.14498E 03	-0.91989E 00
0.16356E 03	0.16496E 03	-0.85732E 00
0.18356E 03	0.18442E 03	-0.46707E 00
0.20336E 03	0.20456E 03	-0.59135E 00
0.23336E 03	0.23454E 03	-0.50387E 00
0.26341E 03	0.26348E 03	-0.27573E-01
0.29331E 03	0.29141E 03	0.64852E 00
0.32346E 03	0.32170E 03	0.54505E 00
0.36346E 03	0.36582E 03	-0.65017E 00
0.40316E 03	0.40310E 03	0.14245E-01
0.45356E 03	0.45152E 03	0.44906E 00
0.53556E 03	0.53608E 03	-0.97393E-01

TABLE D-61

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 600 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = SS-1-A
RUN NUMBER = 2

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.736406E 02	0.135424E 02	0.211662E-03	0.176748E-01	0.938616E 01
0.815599E 02	0.135617E 02	0.235225E-03	0.176990E-01	0.941254E 01
0.816711E 02	0.136477E 02	0.239425E-03	0.176287E-01	0.932556E 01
0.936711E 02	0.136477E 02	0.299219E-03	0.176409E-01	0.933535E 01
0.107661E 03	0.136650E 02	0.376249E-03	0.176372E-01	0.932449E 01
0.123861E 03	0.136650E 02	0.475156E-03	0.176542E-01	0.933839E 01
0.143461E 03	0.136650E 02	0.607951E-03	0.176649E-01	0.934292E 01
0.163655E 03	0.136592E 02	0.758037E-03	0.176819E-01	0.935499E 01
0.193655E 03	0.136610E 02	0.100797E-02	0.176983E-01	0.936200E 01
0.223551E 03	0.136572E 02	0.129058E-02	0.177148E-01	0.936903E 01
0.263551E 03	0.136590E 02	0.172105E-02	0.177369E-01	0.937849E 01
0.303651E 03	0.136590E 02	0.221610E-02	0.177591E-01	0.938807E 01
0.353551E 03	0.136626E 02	0.289769E-02	0.177842E-01	0.939660E 01
0.403851E 03	0.136644E 02	0.369918E-02	0.178124E-01	0.940886E 01
0.463151E 03	0.136662E 02	0.471489E-02	0.178461E-01	0.942353E 01
0.535151E 03	0.136698E 02	0.620806E-02	0.178902E-01	0.944519E 01
0.635651E 03	0.136779E 02	0.869128E-02	0.179488E-01	0.947115E 01
0.780651E 03	0.136933E 02	0.132439E-01	0.180358E-01	0.951008E 01
0.888651E 03	0.137095E 02	0.173755E-01	0.181081E-01	0.954737E 01
0.103465E 04	0.138359E 02	0.253130E-01	0.182017E-01	0.959026E 01
0.117215E 04	0.138937E 02	0.367867E-01	0.182937E-01	0.963284E 01
0.126764E 04	0.139743E 02	0.515854E-01	0.183601E-01	0.966381E 01

TABLE D-62

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.1566E-01	0.4804E 02	0.5375E 14
1	0.4471E-02	0.8310E 02	-0.4562E 14
2	0.7555E-02	0.6226E 02	-0.2171E 14
3	0.9792E-02	0.5994E 02	-0.2608E 14
4	0.1044E-01	0.5734E 02	-0.2401E 14
5	0.1047E-01	0.5735E 02	-0.2405E 14
6	0.1047E-01	0.5735E 02	-0.2405E 14
7	0.1047E-01	0.5735E 02	-0.2405E 14

FRACTIONAL POROSITY = 0.1328E-03

SUM OF ERROR SQUARES = 0.1154E 03
VARIANCE = 0.6077E 01
STANDARD DEVIATION = 0.2465E 01

VARIANCE-COVARIANCE MATRIX

0.30679E-08	-0.64481E-04	0.15250E 08
-0.64481E-04	0.17432E 01	-0.29324E 12
0.15250E 08	-0.29324E 12	0.86230E 23

CORRELATION MATRIX

1.00000	-0.88171	0.93762
-0.88171	1.00000	-0.75635
0.93762	-0.75635	1087.70103

TABLE D-63

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.73640E 02	0.75213E 02	-0.21360E 01
0.81559E 02	0.78511E 02	0.37379E 01
0.81671E 02	0.79241E 02	0.29746E 01
0.93671E 02	0.91553E 02	0.22603E 01
0.10766E 03	0.10588E 03	0.16466E 01
0.12386E 03	0.12289E 03	0.77820E 00
0.14346E 03	0.14329E 03	0.11757E 00
0.16365E 03	0.16405E 03	-0.24096E 00
0.19365E 03	0.19460E 03	-0.48952E 00
0.22355E 03	0.22508E 03	-0.68676E 00
0.26355E 03	0.26582E 03	-0.86354E 00
0.30365E 03	0.30684E 03	-0.10500E 01
0.35355E 03	0.35594E 03	-0.67793E 00
0.40385E 03	0.40677E 03	-0.72455E 00
0.46315E 03	0.46294E 03	0.45688E-01
0.53515E 03	0.53505E 03	0.18932E-01
0.63565E 03	0.63518E 03	0.72859E-01
0.78065E 03	0.77961E 03	0.13303E 00
0.88865E 03	0.88298E 03	0.63788E 00
0.10346E 04	0.10338E 04	0.77018E-01
0.11721E 04	0.11766E 04	-0.38532E 00
0.12676E 04	0.12663E 04	0.10113E 00

TABLE D-64

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 1000 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = SS-1-A
RUN NUMBER = 3

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.936247E 02	0.136265E 02	0.263135E-03	0.177056E-01	0.941540E 01
0.103424E 03	0.136265E 02	0.310692E-03	0.177304E-01	0.944180E 01
0.117432E 03	0.136351E 02	0.381492E-03	0.177493E-01	0.945905E 01
0.133732E 03	0.136360E 02	0.471484E-03	0.177582E-01	0.946294E 01
0.153632E 03	0.136360E 02	0.593158E-03	0.177690E-01	0.946770E 01
0.173632E 03	0.136369E 02	0.727753E-03	0.177800E-01	0.947249E 01
0.203482E 03	0.136378E 02	0.941360E-03	0.177964E-01	0.947968E 01
0.243782E 03	0.136378E 02	0.127990E-02	0.178186E-01	0.948946E 01
0.283032E 03	0.136387E 02	0.167699E-02	0.178428E-01	0.950222E 01
0.335261E 03	0.136687E 02	0.215423E-02	0.178807E-01	0.952620E 01
0.392961E 03	0.136705E 02	0.288280E-02	0.179131E-01	0.954067E 01
0.501661E 03	0.136750E 02	0.447975E-02	0.179856E-01	0.958208E 01
0.612661E 03	0.136813E 02	0.635835E-02	0.180500E-01	0.961149E 01
0.806661E 03	0.136939E 02	0.106136E-01	0.181665E-01	0.966538E 01
0.101567E 04	0.137253E 02	0.167070E-01	0.183126E-01	0.974608E 01
0.122267E 04	0.138228E 02	0.241098E-01	0.184519E-01	0.981313E 01
0.142667E 04	0.138733E 02	0.339633E-01	0.185986E-01	0.988479E 01
0.162267E 04	0.139528E 02	0.473428E-01	0.187498E-01	0.995958E 01
0.193067E 04	0.142020E 02	0.797098E-01	0.190111E-01	0.100903E 02

TABLE D-65

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.8400E-02	0.5850E 02	0.1720E 14
1	0.7731E-02	0.8846E 02	-0.2626E 14
2	0.8180E-02	0.7679E 02	-0.1786E 14
3	0.8257E-02	0.7558E 02	-0.1723E 14
4	0.8257E-02	0.7559E 02	-0.1723E 14
5	0.8257E-02	0.7559E 02	-0.1723E 14

FRACTIONAL POROSITY = 0.1348E-03

SUM OF ERROR SQUARES = 0.3928E 03
VARIANCE = 0.2455E 02
STANDARD DEVIATION = 0.4955E 01

VARIANCE-COVARIANCE MATRIX

0.39572E-08	-0.13790E-03	0.21368E 08
-0.13790E-03	0.65450E 01	-0.66803E 12
0.21368E 08	-0.66803E 12	0.13431E 24

CORRELATION MATRIX

1.00000	-0.85689	0.92686
-0.85689	1.00000	-0.71248
0.92686	-0.71248	419.34885

TABLE D-66

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.93624E 02	0.87566E 02	0.64707E 01
0.10342E 03	0.98913E 02	0.43620E 01
0.11743E 03	0.11397E 03	0.29430E 01
0.13373E 03	0.13133E 03	0.17962E 01
0.15363E 03	0.15272E 03	0.59389E 00
0.17363E 03	0.17419E 03	-0.32432E 00
0.20348E 03	0.20435E 03	-0.43034E 00
0.24378E 03	0.24674E 03	-0.12143E 01
0.28303E 03	0.29068E 03	-0.27031E 01
0.33526E 03	0.33498E 03	0.81796E-01
0.39296E 03	0.39670E 03	-0.95359E 00
0.50166E 03	0.50852E 03	-0.13685E 01
0.61266E 03	0.61496E 03	-0.37565E 00
0.80666E 03	0.80637E 03	0.36106E-01
0.10156E 04	0.10166E 04	-0.96293E-01
0.12226E 04	0.12141E 04	0.69941E 00
0.14266E 04	0.14198E 04	0.48048E 00
0.16226E 04	0.16304E 04	-0.48081E 00
0.19306E 04	0.19301E 04	0.29248E-01

TABLE D-67

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 200 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = SS-2-A
RUN NUMBER = 1A

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.151059E 02	0.136324E 02	0.428399E-03	0.176797E-01	0.941774E 01
0.155971E 02	0.136324E 02	0.574019E-03	0.176853E-01	0.942441E 01
0.160882E 02	0.136324E 02	0.720332E-03	0.176856E-01	0.942453E 01
0.165794E 02	0.136342E 02	0.882683E-03	0.176859E-01	0.942464E 01
0.171100E 02	0.136360E 02	0.105314E-02	0.176903E-01	0.942991E 01
0.175766E 02	0.136273E 02	0.119875E-02	0.176906E-01	0.943001E 01
0.185343E 02	0.136282E 02	0.153174E-02	0.176911E-01	0.943024E 01
0.195167E 02	0.136291E 02	0.187816E-02	0.176916E-01	0.943047E 01
0.209901E 02	0.136300E 02	0.240798E-02	0.176924E-01	0.943081E 01
0.224636E 02	0.136318E 02	0.297746E-02	0.176932E-01	0.943116E 01
0.254106E 02	0.136354E 02	0.420295E-02	0.176948E-01	0.943185E 01
0.283575E 02	0.136390E 02	0.553641E-02	0.176964E-01	0.943254E 01
0.322675E 02	0.136251E 02	0.754306E-02	0.176985E-01	0.943346E 01
0.371791E 02	0.136359E 02	0.102541E-01	0.177012E-01	0.943461E 01
0.430730E 02	0.136468E 02	0.139757E-01	0.177044E-01	0.943600E 01
0.496034E 02	0.137732E 02	0.185566E-01	0.177080E-01	0.943757E 01
0.576889E 02	0.137875E 02	0.250119E-01	0.177143E-01	0.944181E 01
0.675889E 02	0.138363E 02	0.340926E-01	0.177197E-01	0.944416E 01
0.795889E 02	0.139103E 02	0.466839E-01	0.177263E-01	0.944702E 01
0.935889E 02	0.140349E 02	0.638269E-01	0.177340E-01	0.945036E 01
0.107584E 03	0.141944E 02	0.833417E-01	0.177417E-01	0.945372E 01
0.123584E 03	0.144436E 02	0.108869E 00	0.177505E-01	0.945758E 01
0.143684E 03	0.149102E 02	0.145016E 00	0.177618E-01	0.946249E 01
0.163584E 03	0.154996E 02	0.185336E 00	0.177730E-01	0.946739E 01
0.183584E 03	0.163837E 02	0.230640E 00	0.177844E-01	0.947241E 01
0.213584E 03	0.180291E 02	0.306461E 00	0.178018E-01	0.948005E 01
0.243274E 03	0.201560E 02	0.389830E 00	0.178219E-01	0.949093E 01
0.283374E 03	0.238396E 02	0.513341E 00	0.178461E-01	0.950165E 01
0.324764E 03	0.280539E 02	0.649676E 00	0.178968E-01	0.954455E 01
0.374564E 03	0.345372E 02	0.833353E 00	0.179284E-01	0.955873E 01

TABLE D-68

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.3433E 01	0.7058E 01	0.5551E 10
1	0.1937E 01	0.6620E 01	0.7361E 10
2	0.2260E 01	0.6507E 01	0.7391E 10
3	0.2303E 01	0.6521E 01	0.7320E 10
4	0.2304E 01	0.6521E 01	0.7320E 10
5	0.2304E 01	0.6521E 01	0.7320E 10

FRACTIONAL POROSITY = 0.7286E-01

SUM OF ERROR SQUARES = 0.1538E 01
VARIANCE = 0.5698E-01
STANDARD DEVIATION = 0.2387E 00

VARIANCE-COVARIANCE MATRIX

0.63438E-04	-0.15214E-02	0.11361E 07
-0.15214E-02	0.42766E-01	-0.24466E 08
0.11361E 07	-0.24466E 08	0.24058E 17

CORRELATION MATRIX

1.00000	-0.92368	0.91964
-0.92368	1.00000	-0.76276
0.91964	-0.76276	1.00000

TABLE D-69

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.15105E 02	0.15156E 02	-0.33645E 00
0.15597E 02	0.15647E 02	-0.32081E 00
0.16088E 02	0.16124E 02	-0.22504E 00
0.16579E 02	0.16661E 02	-0.49459E 00
0.17110E 02	0.17203E 02	-0.54879E 00
0.17576E 02	0.17635E 02	-0.33655E 00
0.18534E 02	0.18646E 02	-0.60664E 00
0.19516E 02	0.19648E 02	-0.67446E 00
0.20990E 02	0.21097E 02	-0.50968E 00
0.22463E 02	0.22583E 02	-0.53565E 00
0.25410E 02	0.25549E 02	-0.54709E 00
0.28357E 02	0.28488E 02	-0.46190E 00
0.32267E 02	0.32509E 02	-0.75027E 00
0.37179E 02	0.37345E 02	-0.44825E 00
0.43073E 02	0.43245E 02	-0.39935E 00
0.49603E 02	0.49704E 02	-0.20428E 00
0.57688E 02	0.57686E 02	0.42055E-02
0.67588E 02	0.67507E 02	0.12080E 00
0.79588E 02	0.79328E 02	0.32697E 00
0.93588E 02	0.93289E 02	0.32032E 00
0.10758E 03	0.10720E 03	0.35151E 00
0.12358E 03	0.12332E 03	0.21262E 00
0.14368E 03	0.14342E 03	0.17894E 00
0.16358E 03	0.16333E 03	0.15258E 00
0.18358E 03	0.18355E 03	0.16639E-01
0.21358E 03	0.21388E 03	-0.14125E 00
0.24327E 03	0.24385E 03	-0.23705E 00
0.28337E 03	0.28386E 03	-0.17386E 00
0.32476E 03	0.32451E 03	0.76978E-01
0.37456E 03	0.37431E 03	0.65456E-01

TABLE D-70

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 200 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = ARGON
SAMPLE NUMBER = SS-2-A
RUN NUMBER = 1B

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.145664E 02	0.135841E 02	0.216376E-03	0.224130E-01	0.118957E 02
0.155487E 02	0.135868E 02	0.445957E-03	0.224136E-01	0.118960E 02
0.160890E 02	0.135877E 02	0.569149E-03	0.224568E-01	0.119467E 02
0.175394E 02	0.135906E 02	0.927501E-03	0.224578E-01	0.119471E 02
0.185545E 02	0.136010E 02	0.119103E-02	0.224626E-01	0.119522E 02
0.195368E 02	0.136019E 02	0.145284E-02	0.224633E-01	0.119525E 02
0.209611E 02	0.136028E 02	0.185218E-02	0.224643E-01	0.119529E 02
0.224346E 02	0.136046E 02	0.228528E-02	0.224653E-01	0.119533E 02
0.249492E 02	0.136179E 02	0.307295E-02	0.224745E-01	0.119628E 02
0.283873E 02	0.136215E 02	0.424141E-02	0.224768E-01	0.119638E 02
0.332989E 02	0.136287E 02	0.615383E-02	0.224802E-01	0.119653E 02
0.392419E 02	0.136395E 02	0.872955E-02	0.224844E-01	0.119671E 02
0.457034E 02	0.136540E 02	0.119507E-01	0.224929E-01	0.119738E 02
0.537034E 02	0.136756E 02	0.165550E-01	0.224985E-01	0.119762E 02
0.636534E 02	0.137118E 02	0.230194E-01	0.225055E-01	0.119792E 02
0.736034E 02	0.138273E 02	0.306683E-01	0.225126E-01	0.119823E 02
0.876034E 02	0.139284E 02	0.429913E-01	0.225225E-01	0.119866E 02
0.103603E 03	0.140801E 02	0.594220E-01	0.225339E-01	0.119915E 02
0.123703E 03	0.143498E 02	0.834648E-01	0.225444E-01	0.119928E 02
0.143603E 03	0.147182E 02	0.111207E 00	0.225589E-01	0.119991E 02
0.163603E 03	0.152339E 02	0.142301E 00	0.225737E-01	0.120056E 02
0.183603E 03	0.159461E 02	0.176749E 00	0.225887E-01	0.120122E 02
0.213503E 03	0.173950E 02	0.235310E 00	0.226116E-01	0.120222E 02
0.243603E 03	0.192614E 02	0.301052E 00	0.226351E-01	0.120326E 02
0.283603E 03	0.225031E 02	0.397573E 00	0.226671E-01	0.120469E 02
0.323403E 03	0.265060E 02	0.501649E 00	0.227120E-01	0.120777E 02
0.373303E 03	0.324491E 02	0.648449E 00	0.227541E-01	0.120971E 02
0.453603E 03	0.441131E 02	0.922851E 00	0.228243E-01	0.121298E 02
0.545603E 03	0.602131E 02	0.127571E 01	0.229076E-01	0.121691E 02

TABLE D-71

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.3433E 01	0.7058E 01	0.5551E 10
1	0.1635E 01	0.8102E 01	0.5438E 10
2	0.2055E 01	0.8261E 01	0.5108E 10
3	0.2148E 01	0.8188E 01	0.5047E 10
4	0.2152E 01	0.8187E 01	0.5046E 10
5	0.2152E 01	0.8187E 01	0.5046E 10

FRACTIONAL POROSITY = 0.7815E-01

SUM OF ERROR SQUARES = 0.8560E 01
VARIANCE = 0.3292E 00
STANDARD DEVIATION = 0.5738E 00

VARIANCE-COVARIANCE MATRIX

0.12050E-03	-0.39876E-02	0.15366E 07
-0.39876E-02	0.16271E 00	-0.44810E 08
0.15366E 07	-0.44810E 08	0.23998E 17

CORRELATION MATRIX

1.00000	-0.90054	0.90359
-0.90054	1.00000	-0.71708
0.90359	-0.71708	1.00000

TABLE D-72

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14566E 02	0.14544E 02	0.14880E 00
0.15548E 02	0.15534E 02	0.90713E-01
0.16089E 02	0.16048E 02	0.25343E 00
0.17539E 02	0.17482E 02	0.32430E 00
0.18554E 02	0.18501E 02	0.28448E 00
0.19536E 02	0.19460E 02	0.39238E 00
0.20961E 02	0.20866E 02	0.45333E 00
0.22434E 02	0.22318E 02	0.51857E 00
0.24949E 02	0.24815E 02	0.53745E 00
0.28387E 02	0.28178E 02	0.73489E 00
0.33298E 02	0.33132E 02	0.49908E 00
0.39241E 02	0.38965E 02	0.70342E 00
0.45703E 02	0.45407E 02	0.64718E 00
0.53703E 02	0.53471E 02	0.43122E 00
0.63653E 02	0.63264E 02	0.61115E 00
0.73603E 02	0.73426E 02	0.23996E 00
0.87603E 02	0.87559E 02	0.49745E-01
0.10360E 03	0.10372E 03	-0.11951E 00
0.12370E 03	0.12394E 03	-0.19908E 00
0.14360E 03	0.14415E 03	-0.38543E 00
0.16360E 03	0.16416E 03	-0.34352E 00
0.18360E 03	0.18410E 03	-0.27141E 00
0.21350E 03	0.21432E 03	-0.38515E 00
0.24360E 03	0.24446E 03	-0.35478E 00
0.28360E 03	0.28395E 03	-0.12297E 00
0.32340E 03	0.32245E 03	0.29309E 00
0.37330E 03	0.37146E 03	0.49345E 00
0.45360E 03	0.45306E 03	0.11945E 00
0.54560E 03	0.54658E 03	-0.17923E 00

TABLE D-73

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 200 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = METHANE
SAMPLE NUMBER = SS-2-A
RUN NUMBER = 1C

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.145471E 02	0.135647E 02	0.396602E-03	0.111051E-01	0.591241E 01
0.150628E 02	0.135647E 02	0.613402E-03	0.111226E-01	0.593383E 01
0.155294E 02	0.135647E 02	0.813182E-03	0.111051E-01	0.591207E 01
0.160205E 02	0.135647E 02	0.102905E-02	0.111051E-01	0.591190E 01
0.164537E 02	0.135647E 02	0.123882E-02	0.110665E-01	0.586424E 01
0.174360E 02	0.135085E 02	0.170830E-02	0.110665E-01	0.586392E 01
0.184183E 02	0.135103E 02	0.219443E-02	0.110665E-01	0.586358E 01
0.194103E 02	0.135122E 02	0.270271E-02	0.110788E-01	0.587821E 01
0.208838E 02	0.135236E 02	0.350799E-02	0.110788E-01	0.587770E 01
0.223573E 02	0.135254E 02	0.436128E-02	0.110788E-01	0.587719E 01
0.248131E 02	0.135272E 02	0.588049E-02	0.110789E-01	0.587634E 01
0.282608E 02	0.135405E 02	0.816512E-02	0.110843E-01	0.588171E 01
0.331479E 02	0.135477E 02	0.119282E-01	0.110844E-01	0.588003E 01
0.390664E 02	0.135622E 02	0.171468E-01	0.110845E-01	0.587798E 01
0.456261E 02	0.136633E 02	0.236306E-01	0.110864E-01	0.587785E 01
0.535761E 02	0.136886E 02	0.329137E-01	0.110865E-01	0.587511E 01
0.635261E 02	0.137391E 02	0.462136E-01	0.110867E-01	0.587167E 01
0.735261E 02	0.138005E 02	0.618320E-01	0.110870E-01	0.586821E 01
0.875261E 02	0.139233E 02	0.869306E-01	0.110872E-01	0.586336E 01
0.103545E 03	0.141305E 02	0.120645E 00	0.110838E-01	0.585316E 01
0.123635E 03	0.145181E 02	0.169337E 00	0.110739E-01	0.583333E 01
0.143535E 03	0.151074E 02	0.223895E 00	0.110744E-01	0.582629E 01
0.163535E 03	0.159179E 02	0.285686E 00	0.110748E-01	0.581915E 01
0.183235E 03	0.169739E 02	0.351495E 00	0.110753E-01	0.581203E 01
0.213435E 03	0.190858E 02	0.465635E 00	0.110863E-01	0.581385E 01
0.243435E 03	0.217381E 02	0.589098E 00	0.110871E-01	0.580272E 01
0.282935E 03	0.261340E 02	0.766874E 00	0.110881E-01	0.578778E 01
0.323785E 03	0.316595E 02	0.981831E 00	0.110891E-01	0.577204E 01

TABLE D-74

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.3433E 01	0.7058E 01	0.5551E 10
1	0.1807E 01	0.6056E 01	0.8114E 10
2	0.2168E 01	0.5787E 01	0.8284E 10
3	0.2227E 01	0.5837E 01	0.8167E 10
4	0.2228E 01	0.5837E 01	0.8167E 10
5	0.2228E 01	0.5837E 01	0.8167E 10

FRACTIONAL POROSITY = 0.7350E-01

SUM OF ERROR SQUARES = 0.3437E 01
VARIANCE = 0.1374E 00
STANDARD DEVIATION = 0.3707E 00

VARIANCE-COVARIANCE MATRIX

0.21661E-03	-0.49015E-02	0.36844E 07
-0.49015E-02	0.12872E 00	-0.74647E 08
0.36844E 07	-0.74647E 08	0.74438E 17

CORRELATION MATRIX

1.00000	-0.92823	0.91756
-0.92823	1.00000	-0.76257
0.91756	-0.76257	1.00000

TABLE D-75

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14547E 02	0.14506E 02	0.27605E 00
0.15062E 02	0.15012E 02	0.33471E 00
0.15529E 02	0.15453E 02	0.48878E 00
0.16020E 02	0.15928E 02	0.57493E 00
0.16453E 02	0.16363E 02	0.54833E 00
0.17436E 02	0.17281E 02	0.88670E 00
0.18418E 02	0.18258E 02	0.86962E 00
0.19410E 02	0.19254E 02	0.80488E 00
0.20883E 02	0.20749E 02	0.64147E 00
0.22357E 02	0.22239E 02	0.52676E 00
0.24813E 02	0.24709E 02	0.41831E 00
0.28260E 02	0.28091E 02	0.59858E 00
0.33147E 02	0.33033E 02	0.34501E 00
0.39066E 02	0.38998E 02	0.17420E 00
0.45626E 02	0.45485E 02	0.30811E 00
0.53576E 02	0.53543E 02	0.60919E-01
0.63526E 02	0.63494E 02	0.49168E-01
0.73526E 02	0.73658E 02	-0.17938E 00
0.87526E 02	0.87780E 02	-0.29051E 00
0.10354E 03	0.10403E 03	-0.46911E 00
0.12363E 03	0.12408E 03	-0.36543E 00
0.14353E 03	0.14378E 03	-0.17593E 00
0.16353E 03	0.16370E 03	-0.10284E 00
0.18323E 03	0.18297E 03	0.14434E 00
0.21343E 03	0.21342E 03	0.59623E-02
0.24343E 03	0.24297E 03	0.18789E 00
0.28293E 03	0.28176E 03	0.41267E 00
0.32378E 03	0.32477E 03	-0.30405E 00

TABLE D-76

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 400 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = SS-2-A
RUN NUMBER = 2

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.146341E 02	0.136572E 02	0.250206E-03	0.177048E-01	0.944882E 01
0.151252E 02	0.136572E 02	0.378309E-03	0.177050E-01	0.944893E 01
0.156164E 02	0.136572E 02	0.512612E-03	0.177022E-01	0.944515E 01
0.161076E 02	0.136572E 02	0.647680E-03	0.177024E-01	0.944527E 01
0.165987E 02	0.136572E 02	0.783096E-03	0.177027E-01	0.944538E 01
0.175810E 02	0.136572E 02	0.107076E-02	0.177032E-01	0.944561E 01
0.185634E 02	0.136581E 02	0.136925E-02	0.177037E-01	0.944584E 01
0.195457E 02	0.136590E 02	0.167704E-02	0.177043E-01	0.944607E 01
0.210192E 02	0.136608E 02	0.215576E-02	0.176996E-01	0.943971E 01
0.224926E 02	0.136617E 02	0.265531E-02	0.177004E-01	0.944006E 01
0.254887E 02	0.136644E 02	0.377000E-02	0.177021E-01	0.944076E 01
0.294180E 02	0.136680E 02	0.539361E-02	0.177042E-01	0.944169E 01
0.343296E 02	0.136752E 02	0.761051E-02	0.177016E-01	0.943629E 01
0.401744E 02	0.136861E 02	0.107078E-01	0.177047E-01	0.943767E 01
0.476518E 02	0.137023E 02	0.153479E-01	0.177088E-01	0.943943E 01
0.557518E 02	0.137258E 02	0.210126E-01	0.176595E-01	0.937501E 01
0.556357E 02	0.137433E 02	0.207900E-01	0.176879E-01	0.941010E 01
0.656518E 02	0.139136E 02	0.287478E-01	0.176905E-01	0.940890E 01
0.776518E 02	0.139732E 02	0.398744E-01	0.176971E-01	0.941172E 01
0.916518E 02	0.140707E 02	0.547561E-01	0.177184E-01	0.943192E 01
0.107651E 03	0.142224E 02	0.747111E-01	0.177272E-01	0.943574E 01
0.125751E 03	0.144535E 02	0.101102E 00	0.177372E-01	0.944009E 01
0.145651E 03	0.148060E 02	0.133847E 00	0.177482E-01	0.944490E 01
0.167651E 03	0.153708E 02	0.174860E 00	0.177606E-01	0.945029E 01
0.189751E 03	0.161567E 02	0.220273E 00	0.177788E-01	0.946284E 01
0.213451E 03	0.172863E 02	0.273707E 00	0.177925E-01	0.946883E 01
0.243551E 03	0.190791E 02	0.349466E 00	0.178101E-01	0.947656E 01
0.273351E 03	0.213138E 02	0.431297E 00	0.178278E-01	0.948437E 01
0.313751E 03	0.249484E 02	0.551270E 00	0.178523E-01	0.949519E 01
0.363651E 03	0.304248E 02	0.714939E 00	0.178833E-01	0.950896E 01
0.452651E 03	0.428518E 02	0.104933E 01	0.179408E-01	0.953467E 01
0.452651E 03	0.424518E 02	0.104933E 01	0.179406E-01	0.953457E 01

TABLE D-77

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.2304E 01	0.6521E 01	0.7320E 10
1	0.2008E 01	0.7000E 01	0.7852E 10
2	0.2033E 01	0.7029E 01	0.7849E 10
3	0.2033E 01	0.7029E 01	0.7849E 10
4	0.2033E 01	0.7029E 01	0.7849E 10

FRACTIONAL POROSITY = 0.6645E-01

SUM OF ERROR SQUARES = 0.2431E 01
VARIANCE = 0.8383E-01
STANDARD DEVIATION = 0.2895E 00

VARIANCE-COVARIANCE MATRIX

0.39496E-04	-0.12344E-02	0.68948E 06
-0.12344E-02	0.46503E-01	-0.19166E 08
0.68948E 06	-0.19166E 08	0.14327E 17

CORRELATION MATRIX

1.00000	-0.91086	0.91655
-0.91086	1.00000	-0.74253
0.91655	-0.74253	1.00000

TABLE D-78

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14634E 02	0.14652E 02	-0.12246E 00
0.15125E 02	0.15140E 02	-0.99672E-01
0.15616E 02	0.15645E 02	-0.18628E 00
0.16107E 02	0.16139E 02	-0.19945E 00
0.16598E 02	0.16621E 02	-0.13421E 00
0.17581E 02	0.17624E 02	-0.24622E 00
0.18563E 02	0.18623E 02	-0.32382E 00
0.19545E 02	0.19611E 02	-0.33740E 00
0.21019E 02	0.21069E 02	-0.23910E 00
0.22492E 02	0.22514E 02	-0.96331E-01
0.25488E 02	0.25526E 02	-0.14909E 00
0.29418E 02	0.29473E 02	-0.18775E 00
0.34329E 02	0.34219E 02	0.32171E 00
0.40174E 02	0.40113E 02	0.15243E 00
0.47651E 02	0.47797E 02	-0.30641E 00
0.55751E 02	0.55688E 02	0.11323E 00
0.55639E 02	0.55491E 02	0.25964E 00
0.65651E 02	0.65447E 02	0.31121E 00
0.77651E 02	0.77436E 02	0.27706E 00
0.91651E 02	0.91357E 02	0.32123E 00
0.10765E 03	0.10743E 03	0.19909E 00
0.12575E 03	0.12590E 03	-0.12511E 00
0.14565E 03	0.14594E 03	-0.20250E 00
0.16765E 03	0.16811E 03	-0.27790E 00
0.18975E 03	0.19016E 03	-0.21752E 00
0.21345E 03	0.21362E 03	-0.81651E-01
0.24355E 03	0.24378E 03	-0.93914E-01
0.27335E 03	0.27346E 03	-0.40191E-01
0.31375E 03	0.31322E 03	0.16753E 00
0.36365E 03	0.36261E 03	0.28379E 00
0.45265E 03	0.45298E 03	-0.73460E-01
0.45265E 03	0.45293E 03	-0.63320E-01

TABLE D-79.

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 600 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = SS-2-A
RUN NUMBER = 3

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.146824E 02	0.137019E 02	0.244335E-03	0.176515E-01	0.938309E 01
0.151736E 02	0.137019E 02	0.373326E-03	0.176518E-01	0.938320E 01
0.156551E 02	0.136922E 02	0.501399E-03	0.176633E-01	0.939717E 01
0.161102E 02	0.136439E 02	0.638027E-03	0.176354E-01	0.936271E 01
0.165891E 02	0.136439E 02	0.764752E-03	0.176357E-01	0.936281E 01
0.175714E 02	0.136448E 02	0.104292E-02	0.176420E-01	0.937020E 01
0.185537E 02	0.136457E 02	0.132872E-02	0.176393E-01	0.936638E 01
0.200272E 02	0.136475E 02	0.178820E-02	0.176401E-01	0.936672E 01
0.214910E 02	0.136484E 02	0.226101E-02	0.176409E-01	0.936706E 01
0.234556E 02	0.136406E 02	0.292939E-02	0.176419E-01	0.936751E 01
0.254202E 02	0.136424E 02	0.364483E-02	0.176430E-01	0.936797E 01
0.283672E 02	0.136451E 02	0.480185E-02	0.176421E-01	0.936554E 01
0.313142E 02	0.136487E 02	0.604860E-02	0.176437E-01	0.936622E 01
0.352434E 02	0.136541E 02	0.784776E-02	0.176458E-01	0.936713E 01
0.401550E 02	0.136613E 02	0.103602E-01	0.176485E-01	0.936826E 01
0.456324E 02	0.136740E 02	0.135135E-01	0.176515E-01	0.936954E 01
0.537228E 02	0.138214E 02	0.185742E-01	0.177008E-01	0.942684E 01
0.636228E 02	0.138575E 02	0.258089E-01	0.177087E-01	0.943230E 01
0.755802E 02	0.138511E 02	0.364671E-01	0.176312E-01	0.933124E 01
0.895802E 02	0.139396E 02	0.504522E-01	0.176388E-01	0.933447E 01
0.105580E 03	0.140750E 02	0.690410E-01	0.176590E-01	0.935227E 01
0.123780E 03	0.142881E 02	0.936737E-01	0.176690E-01	0.935653E 01
0.143580E 03	0.146203E 02	0.124395E 00	0.176882E-01	0.937142E 01
0.163680E 03	0.150783E 02	0.159683E 00	0.176995E-01	0.937623E 01
0.193580E 03	0.160974E 02	0.219172E 00	0.177164E-01	0.938351E 01
0.223780E 03	0.175586E 02	0.288017E 00	0.177421E-01	0.940126E 01
0.263480E 03	0.200635E 02	0.390983E 00	0.177654E-01	0.941134E 01
0.313264E 03	0.241738E 02	0.530582E 00	0.178604E-01	0.950595E 01
0.353380E 03	0.283887E 02	0.657481E 00	0.178290E-01	0.944632E 01
0.413480E 03	0.356333E 02	0.866394E 00	0.178670E-01	0.946302E 01

TABLE D-80

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.2002E 01	0.7833E 01	0.7037E 10
1	0.1942E 01	0.6922E 01	0.8215E 10
2	0.1944E 01	0.6896E 01	0.8239E 10
3	0.1944E 01	0.6896E 01	0.8239E 10
4	0.1944E 01	0.6896E 01	0.8239E 10

FRACTIONAL POROSITY = 0.6538E-01

SUM OF ERROR SQUARES = 0.3043E 01
VARIANCE = 0.1127E 00
STANDARD DEVIATION = 0.3357E 00

VARIANCE-COVARIANCE MATRIX

0.70623E-04	-0.20939E-02	0.17433E 07
-0.20939E-02	0.74264E-01	-0.46195E 08
0.17433E 07	-0.46195E 08	0.50922E 17

CORRELATION MATRIX

1.00000	-0.91430	0.91930
-0.91430	1.00000	-0.75119
0.91930	-0.75119	1.00000

TABLE D-81

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.14682E 02	0.14721E 02	-0.26545E 00
0.15173E 02	0.15244E 02	-0.46527E 00
0.15655E 02	0.15740E 02	-0.54423E 00
0.16110E 02	0.16213E 02	-0.64356E 00
0.16589E 02	0.16681E 02	-0.55963E 00
0.17571E 02	0.17697E 02	-0.71945E 00
0.18553E 02	0.18688E 02	-0.72805E 00
0.20027E 02	0.20220E 02	-0.96373E 00
0.21491E 02	0.21699E 02	-0.97070E 00
0.23455E 02	0.23656E 02	-0.85744E 00
0.25420E 02	0.25632E 02	-0.83512E 00
0.28367E 02	0.28591E 02	-0.79223E 00
0.31314E 02	0.31526E 02	-0.67913E 00
0.35243E 02	0.35401E 02	-0.44750E 00
0.40155E 02	0.40294E 02	-0.34801E 00
0.45632E 02	0.45813E 02	-0.39631E 00
0.53722E 02	0.53824E 02	-0.18850E 00
0.63622E 02	0.63550E 02	0.11413E 00
0.75580E 02	0.75423E 02	0.20794E 00
0.89580E 02	0.89208E 02	0.41543E 00
0.10558E 03	0.10511E 03	0.43858E 00
0.12378E 03	0.12331E 03	0.37566E 00
0.14358E 03	0.14322E 03	0.24757E 00
0.16368E 03	0.16341E 03	0.16351E 00
0.19358E 03	0.19335E 03	0.11415E 00
0.22378E 03	0.22398E 03	-0.89747E-01
0.26348E 03	0.26434E 03	-0.32933E 00
0.31326E 03	0.31408E 03	-0.26069E 00
0.35338E 03	0.35288E 03	0.13955E 00
0.41348E 03	0.41320E 03	0.67488E-01

TABLE D-82

FLOW RATE PREDICTIONS

USING THE ESTIMATED PARAMETERS K,B,AND FB

WHERE K= 0.1913E 01 MDS
B= 0.7678E 01 PSIA
FB= 0.7179E 10 1/FT

I	QO(OBS.)	QO(PRED.)	PERCENT ERROR
1	0.244335E-03	0.242312E-03	0.827905E 00
2	0.373326E-03	0.367606E-03	0.153198E 01
3	0.501399E-03	0.494850E-03	0.130599E 01
4	0.638027E-03	0.630215E-03	0.122437E 01
5	0.764752E-03	0.760405E-03	0.568367E 00
6	0.104292E-02	0.103512E-02	0.747481E 00
7	0.132872E-02	0.132174E-02	0.524794E 00
8	0.178820E-02	0.177200E-02	0.905438E 00
9	0.226101E-02	0.224298E-02	0.797140E 00
10	0.292939E-02	0.291513E-02	0.486502E 00
11	0.364483E-02	0.362932E-02	0.425311E 00
12	0.480185E-02	0.478290E-02	0.394515E 00
13	0.604860E-02	0.603289E-02	0.259722E 00
14	0.784776E-02	0.785110E-02	-0.425326E-01
15	0.103602E-01	0.103666E-01	-0.624585E-01
16	0.135135E-01	0.134896E-01	0.176361E 00
17	0.185742E-01	0.185737E-01	0.238669E-02
18	0.258089E-01	0.258981E-01	-0.345697E 00
19	0.364671E-01	0.365940E-01	-0.348000E 00
20	0.504522E-01	0.507451E-01	-0.580603E 00
21	0.690410E-01	0.693966E-01	-0.515123E 00
22	0.936737E-01	0.939793E-01	-0.326231E 00
23	0.124395E 00	0.124458E 00	-0.507785E-01
24	0.159683E 00	0.159503E 00	0.112559E 00
25	0.219172E 00	0.218792E 00	0.173261E 00
26	0.288017E 00	0.286650E 00	0.474387E 00
27	0.390983E 00	0.387979E 00	0.768323E 00
28	0.530582E 00	0.528246E 00	0.440253E 00
29	0.657481E 00	0.660595E 00	-0.473659E 00
30	0.866394E 00	0.872038E 00	-0.651486E 00
31	0.143649E 01	0.143360E 01	0.201109E 00

TABLE D-83

INPUT AND PROCESSED DATA

NET CONFINEMENT PRESSURE = 800 PSIG
TEST-METHOD USED = CASE-I
GAS USED AS FLOWING FLUID = NITROGEN
SAMPLE NUMBER = SS-2-A
RUN NUMBER = 4

P(EXT) (PSIA)	P(WELL) (PSIA)	FLOW RATE (MSCF/D)	VISCOSITY (CP)	MU*T*Z (CP*DEG R)
0.150386E 02	0.135460E 02	0.363296E-03	0.177132E-01	0.945903E 01
0.159473E 02	0.135460E 02	0.598334E-03	0.177136E-01	0.945924E 01
0.169296E 02	0.135460E 02	0.860259E-03	0.177142E-01	0.945947E 01
0.174698E 02	0.135460E 02	0.100667E-02	0.177145E-01	0.945960E 01
0.189288E 02	0.135324E 02	0.142446E-02	0.177153E-01	0.945994E 01
0.208935E 02	0.135342E 02	0.202422E-02	0.177163E-01	0.946040E 01
0.233984E 02	0.135369E 02	0.284656E-02	0.177177E-01	0.946099E 01
0.262962E 02	0.135387E 02	0.388719E-02	0.177193E-01	0.946167E 01
0.302500E 02	0.135423E 02	0.544361E-02	0.177214E-01	0.946260E 01
0.351371E 02	0.135495E 02	0.755520E-02	0.177241E-01	0.946376E 01
0.400732E 02	0.135568E 02	0.997334E-02	0.177267E-01	0.946493E 01
0.475261E 02	0.135730E 02	0.141198E-01	0.177283E-01	0.946357E 01
0.575261E 02	0.135983E 02	0.207061E-01	0.177337E-01	0.946594E 01
0.694664E 02	0.137403E 02	0.296823E-01	0.177542E-01	0.948598E 01
0.835164E 02	0.138125E 02	0.424384E-01	0.177618E-01	0.948934E 01
0.995164E 02	0.139281E 02	0.592949E-01	0.177706E-01	0.949318E 01
0.117497E 03	0.140893E 02	0.821585E-01	0.177382E-01	0.944509E 01
0.147497E 03	0.145531E 02	0.127187E 00	0.177548E-01	0.945234E 01
0.173497E 03	0.151916E 02	0.173139E 00	0.177694E-01	0.945871E 01
0.203454E 03	0.162799E 02	0.233655E 00	0.178035E-01	0.948739E 01
0.243204E 03	0.183673E 02	0.327018E 00	0.178265E-01	0.949756E 01
0.283204E 03	0.211669E 02	0.434493E 00	0.178502E-01	0.950805E 01
0.333504E 03	0.256708E 02	0.581147E 00	0.178890E-01	0.953192E 01
0.333404E 03	0.256364E 02	0.570846E 00	0.178889E-01	0.953188E 01
0.393204E 03	0.322425E 02	0.768316E 00	0.179262E-01	0.954861E 01
0.462504E 03	0.413548E 02	0.102592E 01	0.179708E-01	0.956873E 01

TABLE D-84

NUMERICAL PARAMETER ESTIMATION TECHNIQUE

THE GAUSS-NEWTON METHOD
I.E. A NON-LINEAR LEAST SQUARES ALGORITHM

LINEAR SQUARES ESTIMATES

ITER NO	K (MD)	B (PSIA)	FB (1/FT)
IN. GUESS	0.2002E 01	0.7833E 01	0.7037E 10
1	0.1891E 01	0.7007E 01	0.8250E 10
2	0.1895E 01	0.6972E 01	0.8284E 10
3	0.1895E 01	0.6972E 01	0.8284E 10
4	0.1895E 01	0.6972E 01	0.8284E 10

FRACTIONAL POROSITY = 0.6469E-01

SUM OF ERROR SQUARES = 0.1217E 02
VARIANCE = 0.5292E 00
STANDARD DEVIATION = 0.7274E 00

VARIANCE-COVARIANCE MATRIX

0.24787E-03	-0.82328E-02	0.56105E 07
-0.82328E-02	0.33462E 00	-0.16372E 09
0.56105E 07	-0.16372E 09	0.15064E 18

CORRELATION MATRIX

1.00000	-0.90397	0.91814
-0.90397	1.00000	-0.72923
0.91814	-0.72923	1.00000

TABLE D-85

OBS. FUNCTION	CALC. FUNCTION	ERROR (PERCENT)
0.15038E 02	0.15099E 02	-0.40723E 00
0.15947E 02	0.16050E 02	-0.64473E 00
0.16929E 02	0.17059E 02	-0.76606E 00
0.17469E 02	0.17601E 02	-0.75506E 00
0.18928E 02	0.19081E 02	-0.80389E 00
0.20893E 02	0.21087E 02	-0.92985E 00
0.23398E 02	0.23620E 02	-0.94767E 00
0.26296E 02	0.26551E 02	-0.97121E 00
0.30250E 02	0.30503E 02	-0.83827E 00
0.35137E 02	0.35262E 02	-0.35692E 00
0.40073E 02	0.40142E 02	-0.17332E 00
0.47526E 02	0.47506E 02	0.40806E-01
0.57526E 02	0.57543E 02	-0.30198E-01
0.69466E 02	0.69216E 02	0.35964E 00
0.83516E 02	0.83241E 02	0.32979E 00
0.99516E 02	0.99036E 02	0.48217E 00
0.11749E 03	0.11710E 03	0.33413E 00
0.14749E 03	0.14732E 03	0.11642E 00
0.17349E 03	0.17340E 03	0.53402E-01
0.20345E 03	0.20362E 03	-0.83158E-01
0.24320E 03	0.24394E 03	-0.30562E 00
0.28320E 03	0.28476E 03	-0.55247E 00
0.33350E 03	0.33465E 03	-0.34402E 00
0.33340E 03	0.33130E 03	0.63049E 00
0.39320E 03	0.39179E 03	0.35748E 00
0.46250E 03	0.46325E 03	-0.16206E 00

TABLE D-86

A SEMI-GRAPHICAL METHOD OF PARAMETER ESTIMATION
I.E. A SIMULTANEOUS LINEAR LEAST SQUARES FIT TECHNIQUE

NV	B(NV)	KV(NV)	KI(NI)	FB(NI)	NRE(NV)
4	0.8116E 01	0.1854E 01	0.1850E 01	0.6927E 10	0.2138E-03
5	0.8913E 01	0.1791E 01	0.1816E 01	0.6051E 10	0.2597E-03
6	0.8613E 01	0.1814E 01	0.1830E 01	0.6415E 10	0.3834E-03
7	0.8440E 01	0.1827E 01	0.1837E 01	0.6614E 10	0.5438E-03
8	0.8264E 01	0.1841E 01	0.1844E 01	0.6784E 10	0.7450E-03
9	0.8243E 01	0.1842E 01	0.1845E 01	0.6805E 10	0.1019E-02
10	0.8489E 01	0.1824E 01	0.1838E 01	0.6669E 10	0.1346E-02
11	0.8609E 01	0.1815E 01	0.1837E 01	0.6666E 10	0.1730E-02
12	0.8667E 01	0.1811E 01	0.1838E 01	0.6733E 10	0.2398E-02
13	0.8598E 01	0.1816E 01	0.1843E 01	0.6845E 10	0.3463E-02
14	0.8619E 01	0.1814E 01	0.1846E 01	0.6957E 10	0.4898E-02
15	0.8600E 01	0.1816E 01	0.1850E 01	0.7077E 10	0.6945E-02
16	0.8610E 01	0.1815E 01	0.1854E 01	0.7225E 10	0.9691E-02
17	0.8600E 01	0.1816E 01	0.1859E 01	0.7364E 10	0.1346E-01
18	0.8591E 01	0.1816E 01	0.1862E 01	0.7438E 10	0.2057E-01
19	0.8611E 01	0.1815E 01	0.1863E 01	0.7487E 10	0.2778E-01
20	0.8653E 01	0.1812E 01	0.1862E 01	0.7465E 10	0.3684E-01
21	0.8728E 01	0.1807E 01	0.1854E 01	0.7263E 10	0.4942E-01
22	0.8830E 01	0.1800E 01	0.1831E 01	0.6720E 10	0.5999E-01
23	0.9007E 01	0.1789E 01	0.1803E 01	0.6134E 10	0.7214E-01

DATA DISCRIMINATION METHODS

METHOD 1. FINDING THE BEST FIT FOR THE KLINKENBERG PLOT
I.E. THE MAXIMUM CORRELATION COEFFICIENT
NV= 19 R(NV)= 0.999 (MAXIMUM)
NI= 8 R(NI)= 0.988

METHOD 2. MATCHING THE VALUES OF K FROM THE KLINKENBERG AND MODIFIED VISCO-INERTIAL PLOTS
I.E. THE MINIMUM RELATIVE ERROR BETWEEN KV AND KI
NV= 9
NI= 18
ERRK(NV)=-0.1465E 00 (MIN. PERCENT)

METHOD 3. THE REYNOLDS NUMBER CRITERION;I.E. THE POINTS ARE IN VISCOUS-REGION IF NRE IS LESS THAN 0.01
NV= 16
NI= 11
NRE(NV)= 0.9691E-02